

СБОРНИК №2

ЗАДАНИЙ

ПО ВЫСШЕЙ МАТЕМАТИКЕ

для студентов инженерных специальностей

2007

ИНТЕГРАЛЫ

В сборнике подобраны однотипные одинаковой степени сложности задачи по темам. Каждое задание содержит 30 вариантов. Предлагаемые в сборнике задачи предназначены для студентов дневной и заочной форм обучения. Сборник содержит задания по темам, соответствующим программе по курсу высшей математики для студентов инженерных специальностей дневной и заочной формы обучения полного и сокращенного курса.

В сборник включены следующие темы : неопределенные интегралы, определенные интегралы, приложения интегралов, кратные и криволинейные.

Задача 1. Вычислить неопределенный интеграл.

- | | | |
|--|-------------------------------------|--|
| 1. $\int (4 - 3x)e^{-3x} dx.$ | 2. $\int \arctg \sqrt{4x - 1} dx.$ | 3. $\int (3x + 4)e^{3x} dx.$ 1.4. |
| 4. $\int (4x - 2) \cos 2x dx.$ | 5. $\int (4 - 16x) \sin 4x dx.$ | 6. $\int (5x - 2)e^{3x} dx.$ |
| 7. $\int (1 - 6x)e^{2x} dx.$ | 8. $\int \ln(x^2 + 4) dx.$ | 9. $\int \ln(4x^2 + 1) dx.$ |
| 10. $\int (2 - 4x) \sin 2x dx.$ | 11. $\int \arctg \sqrt{6x - 1} dx.$ | 12. $\int e^{-2x} (4x - 3) dx.$ |
| 13. $\int e^{-3x} (2 - 9x) dx.$ | 14. $\int \arctg \sqrt{2x - 1} dx.$ | 15. $\int \arctg \sqrt{3x - 1} dx.$ |
| 16. $\int \arctg \sqrt{5x - 1} dx.$ | 17. $\int (5x + 6) \cos 2x dx.$ | 18. $\int (3x - 2) \cos 5x dx.$ |
| 19. $\int (x\sqrt{2} - 3) \cos 2x dx.$ | 20. $\int (4x + 7) \cos 3x dx.$ | 21. $\int (2x - 5) \cos 4x dx.$ |
| 22. $\int (8 - 3x) \cos 5x dx.$ | 23. $\int (x + 5) \sin 3x dx.$ | 24. $\int (2 - 3x) \sin 2x dx.$ |
| 25. $\int (4x + 3) \sin 5x dx.$ | 26. $\int (7x - 10) \sin 4x dx.$ | 27. $\int (\sqrt{2} - 8x) \sin 3x dx.$ |
| 28. $\int \frac{xdx}{\cos^2 x}.$ | 29. $\int \frac{xdx}{\sin^2 x}.$ | 30. $\int x \sin^2 x dx.$ |

Задача 2. Вычислить определенные интегралы.

$$1. \int_{-2}^0 (x^2 + 5x + 6) \cos 2x dx.$$

$$2. \int_{-2}^0 (x^2 - 4) \cos 3x dx.$$

$$3. \int_{-1}^0 (x^2 + 4x + 3) \cos x dx.$$

$$4. \int_{-2}^0 (x + 2)^2 \cos 3x dx.$$

$$5. \int_{-4}^0 (x^2 + 7x + 12) \cos x dx.$$

$$6. \int_0^{\pi} (2x^2 + 4x + 7) \cos 2x dx.$$

$$7. \int_0^{\pi} (9x^2 + 9x + 11) \cos 3x dx.$$

$$8. \int_0^{\pi} (8x^2 + 16x + 17) \cos 4x dx.$$

$$9. \int_0^{2\pi} (3x^2 + 5) \cos 2x dx.$$

$$10. \int_0^{2\pi} (2x^2 - 15) \cos 3x dx.$$

$$11. \int_0^{2\pi} (3 - 7x^2) \cos 2x dx.$$

$$12. \int_0^{2\pi} (1 - 8x^2) \cos 4x dx.$$

$$13. \int_{-1}^0 (x^2 + 2x + 1) \sin 3x dx.$$

$$14. \int_0^3 (x^2 - 3x) \sin 2x dx.$$

$$15. \int_0^{\pi} (x^2 - 3x + 2) \sin x dx.$$

$$16. \int_0^{\frac{\pi}{2}} (x^2 - 5x + 6) \sin 3x dx.$$

$$17. \int_{-3}^0 (x^2 + 6x + 9) \sin 2x dx.$$

$$18. \int_0^{\frac{\pi}{4}} (x^2 + 17,5) \sin 2x dx.$$

$$19. \int_0^{\frac{\pi}{2}} (1 - 5x^2) \sin x dx.$$

$$20. \int_{\frac{\pi}{4}}^3 (3x - x^2) \sin 2x dx.$$

$$21. \int_1^2 x \ln^2 x dx.$$

$$22. \int_1^{e^2} \frac{\ln^2 x dx}{\sqrt{x}}.$$

$$23. \int_1^8 \frac{\ln^2 x dx}{\sqrt[3]{x^2}}.$$

$$24. \int_0^1 (x + 1) \ln^2 (x + 1) dx.$$

$$25. \int_2^3 (x - 1)^3 \ln^2 (x - 1) dx.$$

$$26. \int_{-1}^0 (x + 2)^3 \ln^2 (x + 2) dx.$$

$$27. \int_0^2 (x + 1)^2 \ln^2 (x + 1) dx.$$

$$28. \int_1^e \sqrt{x} \ln^2 x dx.$$

$$29. \int_{-1}^1 x^2 e^{-\frac{x}{2}} dx.$$

$$30. \int_0^1 x^2 e^{3x} dx.$$

Задача 3. Найти неопределенные интегралы.

$$1. \int \frac{dx}{x\sqrt{x^2 + 1}}.$$

$$2. \int \frac{1 + \ln x}{x} dx.$$

$$3. \int \frac{dx}{x\sqrt{x^2 - 1}}.$$

$$4. \int \frac{x^2 + \ln x^2}{x} dx.$$

$$5. \int \frac{xdx}{\sqrt{x^4 + x^2 + 1}}.$$

$$6. \int \frac{(\arccos x)^3 - 1}{\sqrt{1 - x^2}} dx.$$

$$7. \int \operatorname{tg} x \ln \cos x dx.$$

$$8. \int \frac{\operatorname{tg}(x + 1)}{\cos^2(x + 1)} dx.$$

$$9. \int \frac{x^3}{(x^2 + 1)^2} dx.$$

$$\begin{array}{lll}
10. \int \frac{1 - \cos x}{(x - \sin x)^3} dx. & 11. \int \frac{\sin x - \cos x}{(\cos x + \sin x)^5} dx. & 12. \int \frac{x \cos x + \sin x}{(x \sin x)^2} dx. \\
13. \int \frac{x^3 + x}{x^4 + 1} dx. & 14. \int \frac{x dx}{\sqrt{x^4 - x^2 - 1}}. & 15. \int \frac{x dx}{\sqrt[3]{x - 1}}. \\
16. \int \frac{1 + \ln(x - 1)}{x - 1} dx. & 17. \int \frac{(x^2 + 1) dx}{(x^3 + 3x + 1)^5}. & 18. \int \frac{4 \operatorname{arctg} x - x}{1 + x^2} dx. \\
19. \int \frac{x^3}{x^2 + 4} dx. & 20. \int \frac{x + \cos x}{x^2 + 2 \sin x} dx. & 21. \int \frac{2 \cos x + 3 \sin x}{(2 \sin x - 3 \cos x)^3} dx. \\
22. \int \frac{8x - \operatorname{arctg} 2x}{1 + 4x^2} dx. & 23. \int \frac{1/(2\sqrt{x}) + 1}{(\sqrt{x} + x)^2} dx. & 24. \int \frac{x}{x^4 + 1} dx. \\
25. \int \frac{x + 1/x}{\sqrt{x^2 + 1}} dx. & 26. \int \frac{x - 1/x}{\sqrt{x^2 + 1}} dx. & 27. \int \frac{\operatorname{arctg} x + x}{1 + x^2} dx. \\
28. \int \frac{x - (\operatorname{arctg} x)^4}{1 + x^2} dx. & 29. \int \frac{x^3}{x^2 + 1} dx. & 30. \int \frac{(\arcsin x)^2 + 1}{\sqrt{1 - x^2}} dx.
\end{array}$$

Задача 4. Вычислить определенные интегралы.

$$\begin{array}{lll}
1. \int_{e+1}^{e^2+1} \frac{1 + \ln(x - 1)}{x - 1} dx. & 2. \int_0^1 \frac{(x^2 + 1) dx}{(x^3 + 3x + 1)^2}. & 3. \int_0^1 \frac{4 \operatorname{arctg} x - x}{1 + x^2} dx. \\
4. \int_0^2 \frac{x^3 dx}{x^2 + 4}. & 5. \int_{\pi}^{2\pi} \frac{x + \cos x}{x^2 + 2 \sin x} dx. & 6. \int_0^{\pi/4} \frac{2 \cos x + 3 \sin x}{(2 \sin x - 3 \cos x)^3} dx. \\
7. \int_0^{1/2} \frac{8x - \operatorname{arctg} 2x}{1 + 4x^2} dx. & 8. \int_1^4 \frac{1/(2\sqrt{x}) + 1}{(\sqrt{x} + x)^2} dx. & 9. \int_0^1 \frac{x dx}{x^4 + 1}. \\
10. \int_{\sqrt{3}}^{\sqrt{8}} \frac{x + 1/x}{\sqrt{x^2 + 1}} dx. & 11. \int_{\sqrt{3}}^{\sqrt{8}} \frac{x - 1/x}{\sqrt{x^2 + 1}} dx. & 12. \int_0^{\sqrt{3}} \frac{\operatorname{arctg} x + x}{1 + x^2} dx. \\
13. \int_0^{\sqrt{3}} \frac{x - (\operatorname{arctg} x)^4}{1 + x^2} dx. & 14. \int_0^1 \frac{x^3}{x^2 + 1} dx. & 15. \int_0^{\sin 1} \frac{(\arcsin x)^2 + 1}{\sqrt{1 - x^2}} dx. \\
16. \int_1^3 \frac{1 - \sqrt{x}}{\sqrt{x}(x + 1)} dx. & 17. \int_{\sqrt{3}}^{\sqrt{8}} \frac{dx}{\sqrt{x^2 + 1}}. & 18. \int_1^e \frac{1 + \ln x}{x} dx.
\end{array}$$

$$19. \int_{\sqrt{2}}^2 \frac{dx}{\sqrt{x^2+1}}$$

$$20. \int_1^e \frac{x^2 + \ln x^2}{x} dx.$$

$$21. \int_0^1 \frac{xdx}{\sqrt{x^4+x^2+1}}$$

$$22. \int_0^1 \frac{x^3 dx}{(x^2+1)^2}$$

$$23. \int_0^{\pi/4} \operatorname{tg} x \ln \cos x dx.$$

$$24. \int_{-1}^0 \frac{\operatorname{tg}(x+1)}{\cos^2(x+1)} dx.$$

$$25. \int_0^{1/\sqrt{2}} \frac{(\arccos x)^3 - 1}{\sqrt{1-x^2}} dx.$$

$$26. \int_{\pi}^{2\pi} \frac{1 - \cos x}{(x - \sin x)^2} dx.$$

$$27. \int_0^{\pi/4} \frac{\sin x - \cos x}{(\cos x + \sin x)^5} dx.$$

$$28. \int_{\pi/4}^{\pi/2} \frac{x \cos x + \sin x}{(x \sin x)^2} dx.$$

$$29. \int_0^1 \frac{x^3 + x}{x^4 + 1} dx.$$

$$30. \int_{\sqrt{2}}^{\sqrt{3}} \frac{xdx}{\sqrt{x^4 - x^2 - 1}}$$

Задача 5. Найти неопределенные интегралы.

$$1. \int \frac{x^3 + 1}{x^2 - x} dx.$$

$$2. \int \frac{3x^3 + 1}{x^2 - 1} dx.$$

$$3. \int \frac{x^3 - 17}{x^2 - 4x + 3} dx.$$

$$4. \int \frac{2x^3 + 5}{x^2 - x - 2} dx.$$

$$5. \int \frac{2x^3 - 1}{x^2 + x - 6} dx.$$

$$6. \int \frac{3x^3 + 25}{x^2 + 3x + 2} dx.$$

$$7. \int \frac{x^3 + 2x^2 + 3}{(x-1)(x-2)(x-3)} dx.$$

$$8. \int \frac{3x^3 + 2x^2 + 1}{(x+2)(x-2)(x-1)} dx.$$

$$9. \int \frac{x^3}{(x-1)(x+1)(x+2)} dx.$$

$$10. \int \frac{x^3 - 3x^2 - 12}{(x-4)(x-3)(x-2)} dx.$$

$$11. \int \frac{x^3 - 3x^2 - 12}{(x-4)(x-3)x} dx.$$

$$12. \int \frac{4x^3 + x^2 + 2}{x(x-1)(x-2)} dx.$$

$$13. \int \frac{3x^3 - 2}{x^3 - x} dx.$$

$$14. \int \frac{x^3 - 3x^2 - 12}{(x-4)(x-2)x} dx.$$

$$15. \int \frac{x^5 - x^3 + 1}{x^2 - x} dx.$$

$$16. \int \frac{x^5 + 3x^3 - 1}{x^2 + x} dx.$$

$$17. \int \frac{2x^5 - 8x^3 + 3}{x^2 - 2x} dx.$$

$$18. \int \frac{3x^5 - 12x^3 - 7}{x^2 + 2x} dx.$$

$$19. \int \frac{-x^5 + 9x^3 + 4}{x^2 + 3x} dx.$$

$$20. \int \frac{-x^5 + 25x^3 + 1}{x^2 + 5x} dx.$$

$$21. \int \frac{x^3 - 5x^2 + 5x + 23}{(x-1)(x+1)(x-5)} dx.$$

$$22. \int \frac{x^5 + 2x^4 - 2x^3 + 5x^2 - 7x + 9}{(x+3)(x-1)x} dx.$$

$$23. \int \frac{2x^4 - 5x^2 - 8x - 8}{x(x-2)(x+2)} dx.$$

$$24. \int \frac{4x^4 + 2x^2 - x - 3}{x(x-1)(x+1)} dx.$$

$$25. \int \frac{3x^4 + 3x^3 - 5x^2 + 2}{x(x-1)(x+2)} dx.$$

$$26. \int \frac{2x^4 + 2x^3 - 41x^2 + 20}{x(x-4)(x+5)} dx.$$

27.
$$\int \frac{x^5 - x^4 - 6x^3 + 13x + 6}{x(x-3)(x+2)} dx.$$

28.
$$\int \frac{3x^3 - x^2 - 12x - 2}{x(x+1)(x-2)} dx.$$

29.
$$\int \frac{2x^4 + 2x^3 - 3x^2 + 2x - 9}{x(x-1)(x+3)} dx.$$

30.
$$\int \frac{2x^3 - x^2 - 7x - 12}{x(x-3)(x+1)} dx.$$

Задача 6. Найти неопределенные интегралы.

1.
$$\int \frac{x^3 + 6x^2 + 13x + 9}{(x+1)(x+2)^3} dx.$$

2.
$$\int \frac{x^3 + 6x^2 + 13x + 8}{x(x+2)^3} dx.$$

3.
$$\int \frac{x^3 - 6x^2 + 13x - 6}{(x+2)(x-2)^3} dx.$$

4.
$$\int \frac{x^3 + 6x^2 + 14x + 10}{(x+1)(x+2)^3} dx.$$

5.
$$\int \frac{x^3 - 6x^2 + 11x - 10}{(x+2)(x-2)^3} dx.$$

6.
$$\int \frac{x^3 + 6x^2 + 11x + 7}{(x+1)(x+2)^3} dx.$$

7.
$$\int \frac{2x^3 + 6x^2 + 7x + 1}{(x-1)(x+1)^3} dx.$$

8.
$$\int \frac{x^3 + 6x^2 + 10x + 10}{(x-1)(x+2)^3} dx.$$

9.
$$\int \frac{2x^3 + 6x^2 + 7x + 2}{x(x+1)^3} dx.$$

10.
$$\int \frac{x^3 - 6x^2 + 13x - 8}{x(x-2)^3} dx.$$

11.
$$\int \frac{x^3 - 6x^2 + 13x - 7}{(x+1)(x-2)^3} dx.$$

12.
$$\int \frac{x^3 - 6x^2 + 14x - 6}{(x+1)(x-2)^3} dx.$$

13.
$$\int \frac{x^3 - 6x^2 + 10x - 10}{(x+1)(x-2)^3} dx.$$

14.
$$\int \frac{x^3 + x + 2}{(x+2)x^3} dx.$$

15.
$$\int \frac{3x^3 + 9x^2 + 10x + 2}{(x-1)(x+1)^3} dx.$$

16.
$$\int \frac{2x^3 + x + 1}{(x+1)x^3} dx.$$

17.
$$\int \frac{2x^3 + 6x^2 + 7x + 4}{(x+2)(x+1)^3} dx.$$

18.
$$\int \frac{2x^3 + 6x^2 + 5x}{(x+2)(x+1)^3} dx.$$

19.
$$\int \frac{2x^3 + 6x^2 + 7x}{(x-2)(x+1)^3} dx.$$

20.
$$\int \frac{2x^3 + 6x^2 + 5x + 4}{(x-2)(x+1)^3} dx.$$

21.
$$\int \frac{x^3 + 6x^2 + 4x + 24}{(x-2)(x+2)^3} dx.$$

22.
$$\int \frac{x^3 + 6x^2 + 14x + 4}{(x-2)(x+2)^3} dx.$$

23.
$$\int \frac{x^3 + 6x^2 + 18x - 4}{(x-2)(x+2)^3} dx.$$

24.
$$\int \frac{x^3 + 6x^2 + 10x + 12}{(x-2)(x+2)^3} dx.$$

25.
$$\int \frac{x^3 - 6x^2 + 14x - 4}{(x+2)(x-2)^3} dx.$$

26.
$$\int \frac{x^3 + 6x^2 + 15x + 2}{(x-2)(x+2)^3} dx.$$

27.
$$\int \frac{2x^3 - 6x^2 + 7x - 4}{(x-2)(x-1)^3} dx.$$

28.
$$\int \frac{2x^3 - 6x^2 + 7x}{(x+2)(x-1)^3} dx.$$

29.
$$\int \frac{x^3 + 6x^2 - 10x + 52}{(x-2)(x+2)^3} dx.$$

30.
$$\int \frac{x^3 - 6x^2 + 13x - 6}{(x+2)(x-2)^3} dx.$$

Задача 7. Найти неопределенные интегралы.

1.
$$\int \frac{x^3 + 4x^2 + 4x + 2}{(x+1)^2(x^2 + x + 1)} dx.$$

2.
$$\int \frac{x^3 + 4x^2 + 3x + 2}{(x+1)^2(x^2 + 1)} dx.$$

3.
$$\int \frac{2x^3 + 7x^2 + 7x - 1}{(x+2)^2(x^2 + x + 1)} dx.$$

4.
$$\int \frac{2x^3 + 4x^2 + 2x - 1}{(x+1)^2(x^2 + 2x + 2)} dx.$$

5.
$$\int \frac{x^3 + 6x^2 + 9x + 6}{(x+1)^2(x^2 + 2x + 2)} dx.$$

6.
$$\int \frac{2x^3 + 11x^2 + 16x + 10}{(x+2)^2(x^2 + 2x + 3)} dx.$$

7.
$$\int \frac{3x^3 + 6x^2 + 5x - 1}{(x+1)^2(x^2 + 2)} dx.$$

8.
$$\int \frac{x^3 + 9x^2 + 21x + 21}{(x+3)^2(x^2 + 3)} dx.$$

9.
$$\int \frac{x^3 + 6x^2 + 8x + 8}{(x+2)^2(x^2 + 4)} dx.$$

10.
$$\int \frac{x^3 + 5x^2 + 12x + 4}{(x+2)^2(x^2 + 4)} dx.$$

11.
$$\int \frac{2x^3 - 4x^2 - 16x - 12}{(x-1)^2(x^2 + 4x + 5)} dx.$$

12.
$$\int \frac{-3x^3 + 13x^2 - 13x + 1}{(x-2)^2(x^2 - x + 1)} dx.$$

13.
$$\int \frac{x^3 + 2x^2 + 10x}{(x+1)^2(x^2 - x + 1)} dx.$$

14.
$$\int \frac{3x^3 + x + 46}{(x-1)^2(x^2 + 9)} dx.$$

15.
$$\int \frac{4x^3 + 24x^2 + 20x - 28}{(x+3)^2(x^2 + 2x + 2)} dx.$$

16.
$$\int \frac{2x^3 + 3x^2 + 3x + 2}{(x^2 + x + 1)(x^2 + 1)} dx.$$

17.
$$\int \frac{x^3 + x + 1}{(x^2 + x + 1)(x^2 + 1)} dx.$$

18.
$$\int \frac{x^2 + x + 3}{(x^2 + x + 1)(x^2 + 1)} dx.$$

19.
$$\int \frac{2x^3 + 4x^2 + 2x + 2}{(x^2 + x + 1)(x^2 + x + 2)} dx.$$

20.
$$\int \frac{2x^3 + 7x^2 + 7x + 9}{(x^2 + x + 1)(x^2 + x + 2)} dx.$$

21.
$$\int \frac{4x^2 + 3x + 4}{(x^2 + 1)(x^2 + x + 1)} dx.$$

22.
$$\int \frac{3x^3 + 4x^2 + 6x}{(x^2 + 2)(x^2 + 2x + 2)} dx.$$

23.
$$\int \frac{2x^2 - x + 1}{(x^2 - x + 1)(x^2 + 1)} dx.$$

24.
$$\int \frac{x^3 + x^2 + 1}{(x^2 - x + 1)(x^2 + 1)} dx.$$

$$25. \int \frac{x^3 + x + 1}{(x^2 - x + 1)(x^2 + 1)} dx.$$

$$26. \int \frac{2x^3 + 2x + 1}{(x^2 - x + 1)(x^2 + 1)} dx.$$

$$27. \int \frac{x^3 + 2x^2 + x + 1}{(x^2 + x + 1)(x^2 + 1)} dx.$$

$$28. \int \frac{x + 4}{(x^2 + x + 2)(x^2 + 2)} dx.$$

$$29. \int \frac{2x^3 + 2x^2 + 2x + 1}{(x^2 + x + 1)(x^2 + 1)} dx.$$

$$30. \int \frac{3x^3 + 7x^2 + 12x + 6}{(x^2 + x + 3)(x^2 + 2x + 3)} dx.$$

Задача 8. Вычислить определенные интегралы.

$$1. \int_{\pi/2}^{2 \operatorname{arctg} 2} \frac{dx}{\sin^2 x (1 - \cos x)}.$$

$$2. \int_0^{\pi/2} \frac{\cos x dx}{2 + \cos x}.$$

$$3. \int_{\pi/2}^{2 \operatorname{arctg} 2} \frac{dx}{\sin^2 x (1 + \cos x)}.$$

$$4. \int_{2 \operatorname{arctg}(1/2)}^{\pi/2} \frac{\cos x dx}{(1 - \cos x)^3}.$$

$$5. \int_0^{\pi/2} \frac{\cos x - \sin x}{(1 + \sin x)^2} dx.$$

$$6. \int_{2 \operatorname{arctg} 2}^{2 \operatorname{arctg} 3} \frac{dx}{\cos x (1 - \cos x)}.$$

$$7. \int_{2 \operatorname{arctg}(1/3)}^{2 \operatorname{arctg}(1/2)} \frac{dx}{\sin x (1 - \sin x)}.$$

$$8. \int_{2 \operatorname{arctg}(1/2)}^{\pi/2} \frac{dx}{(1 + \sin x - \cos x)^2}.$$

$$9. \int_0^{\pi/2} \frac{\cos x dx}{5 + 4 \cos x}.$$

$$10. \int_0^{2\pi/3} \frac{1 + \sin x}{1 + \cos x + \sin x} dx.$$

$$11. \int_{\pi/3}^{\pi/2} \frac{\cos x dx}{1 + \sin x - \cos x}.$$

$$12. \int_0^{\pi/2} \frac{(1 + \cos x) dx}{1 + \sin x + \cos x}.$$

$$13. \int_0^{\pi/2} \frac{\sin x dx}{1 + \sin x + \cos x}.$$

$$14. \int_0^{2 \operatorname{arctg}(1/2)} \frac{1 + \sin x}{(1 - \sin x)^2} dx.$$

$$15. \int_0^{\pi/2} \frac{\cos x dx}{1 + \sin x + \cos x}.$$

$$16. \int_0^{2 \operatorname{arctg}(1/3)} \frac{\cos x dx}{(1 - \sin x)(1 + \cos x)}.$$

$$17. \int_{-2\pi/3}^0 \frac{\cos x dx}{1 + \cos x - \sin x}.$$

$$18. \int_{-\pi/2}^0 \frac{\cos x dx}{(1 + \cos x - \sin x)^2}.$$

$$19. \int_0^{\pi/2} \frac{\cos x dx}{(1 + \cos x + \sin x)^2}.$$

$$20. \int_0^{2 \operatorname{arctg}(1/2)} \frac{(1 - \sin x) dx}{\cos x (1 + \cos x)}.$$

$$21. \int_0^{\pi/2} \frac{\sin x dx}{(1 + \sin x)^2}.$$

$$22. \int_0^{\pi/2} \frac{\sin x dx}{(1 + \cos x + \sin x)^2}.$$

$$23. \int_{-\pi/2}^0 \frac{\sin x dx}{(1 + \cos x - \sin x)^2}.$$

$$24. \int_{-2\pi/3}^0 \frac{\cos^2 x dx}{(1 + \cos x - \sin x)^2}.$$

$$25. \int_0^{\pi/2} \frac{\sin^2 x dx}{(1 + \cos x + \sin x)^2}.$$

$$26. \int_0^{2\pi/3} \frac{\cos^2 x dx}{(1 + \cos x - \sin x)^2}.$$

$$27. \int_{\pi/2}^{2\operatorname{arctg} 2} \frac{dx}{\sin x (1 + \sin x)}.$$

$$28. \int_0^{\pi/2} \frac{dx}{(1 + \cos x + \sin x)^2}.$$

$$29. \int_0^{\pi/2} \frac{\sin x dx}{2 + \sin x}.$$

$$30. \int_0^{\pi/4} \frac{dx}{\cos x (1 + \cos x)}.$$

Задача 9. Вычислить определенные интегралы.

$$1. \int_{\pi/2}^{\pi} 2^8 \sin^8 x dx.$$

$$2. \int_0^{\pi} 2^4 \sin^6 x \cos^2 x dx.$$

$$3. \int_0^{2\pi} \sin^4 x \cos^4 x dx.$$

$$4. \int_0^{2\pi} \sin^2(x/4) \cos^6(x/4) dx.$$

$$5. \int_0^{\pi} 2^4 \cos^8(x/2) dx.$$

$$6. \int_{-\pi/2}^0 2^8 \sin^8 x dx.$$

$$7. \int_{\pi/2}^{\pi} 2^4 \sin^6 x \cos^2 x dx.$$

$$8. \int_0^{\pi} 2^4 \sin^4 x \cos^4 x dx.$$

$$9. \int_0^{2\pi} \sin^2 x \cos^6 x dx.$$

$$9. \int_0^{2\pi} \cos^8(x/4) dx.$$

$$11. \int_0^{\pi} 2^4 \sin^8(x/2) dx.$$

$$12. \int_{-\pi}^0 2^8 \sin^6 x \cos^2 x dx.$$

$$13. \int_{\pi/2}^{2\pi} 2^8 \sin^4 x \cos^4 x dx.$$

$$14. \int_0^{\pi} 2^4 \sin^2 x \cos^6 x dx.$$

$$15. \int_0^{2\pi} \cos^8 x dx.$$

$$16. \int_0^{2\pi} \sin^8(x/4) dx.$$

$$17. \int_0^{\pi} 2^4 \sin^6(x/2) \cos^2(x/2) dx.$$

$$18. \int_{-\pi/2}^0 2^8 \sin^4 x \cos^4 x dx.$$

$$19. \int_{\pi/2}^{\pi} 2^8 \sin^2 x \cos^6 x dx.$$

$$20. \int_0^{\pi} 2^4 \cos^8 x dx.$$

21. $\int_0^{2\pi} \sin^8 x \, dx.$

22. $\int_0^{2\pi} \sin^6(x/4) \cos^2(x/4) \, dx.$

23. $\int_0^{\pi} 2^4 \sin^4(x/2) \cos^4(x/2) \, dx.$

24. $\int_{-\pi/2}^0 2^8 \sin^2 x \cos^6 x \, dx.$

25. $\int_{\pi/2}^{2\pi} 2^8 \cos^8 x \, dx.$

26. $\int_0^{\pi} 2^4 \sin^8 x \, dx.$

27. $\int_0^{2\pi} \sin^6 x \cos^2 x \, dx.$

28. $\int_0^{2\pi} \sin^4(x/4) \cos^4(x/4) \, dx.$

29. $\int_0^{\pi} 2^4 \sin^2(x/2) \cos^6(x/2) \, dx.$

30. $\int_{-\pi/2}^0 2^8 \cos^8 x \, dx.$

Задача 10. Вычислить определенные интегралы.

1. $\int_0^{16} \sqrt{256 - x^2} \, dx.$

2. $\int_0^1 x^2 \sqrt{1 - x^2} \, dx.$

3. $\int_0^5 \frac{dx}{(25 + x^2) \sqrt{25 + x^2}}.$

4. $\int_0^3 \frac{dx}{(9 + x^2)^{3/2}}.$

5. $\int_0^{\sqrt{5}/2} \frac{dx}{\sqrt{(5 - x^2)^3}}.$

6. $\int_1^2 \frac{\sqrt{x^2 - 1}}{x^4} \, dx.$

7. $\int_0^{\sqrt{2}/2} \frac{x^4 dx}{\sqrt{(1 - x^2)^3}}.$

8. $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{(4 - x^2)^3}}.$

9. $\int_0^1 \frac{x^4 dx}{(2 - x^2)^{3/2}}.$

10. $\int_0^2 \frac{x^2 dx}{\sqrt{16 - x^2}}.$

11. $\int_0^2 \sqrt{4 - x^2} \, dx.$

12. $\int_0^4 \frac{dx}{(16 + x^2)^{3/2}}.$

13. $\int_0^4 x^2 \sqrt{16 - x^2} \, dx.$

14. $\int_0^{5/2} \frac{x^2 dx}{\sqrt{25 - x^2}}.$

15. $\int_0^5 x^2 \sqrt{25 - x^2} \, dx.$

16. $\int_0^4 \sqrt{16 - x^2} \, dx.$

17. $\int_0^{4\sqrt{3}} \frac{dx}{\sqrt{(64 - x^2)^3}}.$

18. $\int_{\sqrt{2}}^{2\sqrt{2}} \frac{\sqrt{x^2 - 2}}{x^4} \, dx.$

19. $\int_0^{2\sqrt{2}} \frac{x^4 dx}{(16 - x^2) \sqrt{16 - x^2}}.$

20. $\int_{-3}^3 x^2 \sqrt{9 - x^2} \, dx.$

21. $\int_1^{\sqrt{3}} \frac{dx}{\sqrt{(1 + x^2)^3}}.$

22. $\int_0^2 \frac{dx}{\sqrt{(16 - x^2)^3}}.$

23. $\int_0^2 \frac{x^4 dx}{\sqrt{(8 - x^2)^3}}.$

24. $\int_3^6 \frac{\sqrt{x^2 - 9}}{x^4} \, dx.$

25. $\int_0^1 \sqrt{4-x^2} dx.$

26. $\int_2^4 \frac{\sqrt{x^2-4}}{x^4} dx.$

27. $\int_0^2 \frac{dx}{(4+x^2)\sqrt{4+x^2}}.$

28. $\int_0^{\sqrt{2}} \frac{x^4 dx}{(4-x^2)^{3/2}}.$

29. $\int_0^{1/\sqrt{2}} \frac{dx}{(1-x^2)\sqrt{1-x^2}}.$

30. $\int_0^1 \frac{x^2 dx}{\sqrt{4-x^2}}.$

Задача 11. Найти неопределенные интегралы.

1. $\int \frac{\sqrt{1+\sqrt{x}}}{x^4 \sqrt{x^3}} dx.$

2. $\int \frac{\sqrt[3]{1+\sqrt{x}}}{x^3 \sqrt{x^2}} dx.$

3. $\int \frac{\sqrt{1+\sqrt[3]{x}}}{x \sqrt{x}} dx.$

4. $\int \frac{\sqrt[3]{1+\sqrt[3]{x}}}{x^9 \sqrt{x^4}} dx.$

5. $\int \frac{\sqrt[3]{1+\sqrt[3]{x^2}}}{x^9 \sqrt{x^8}} dx.$

6. $\int \frac{\sqrt[3]{(1+\sqrt[3]{x})^2}}{x^9 \sqrt{x^5}} dx.$

7. $\int \frac{\sqrt[3]{(1+\sqrt[3]{x^2})^2}}{x^2 \sqrt{x}} dx.$

8. $\int \frac{\sqrt[3]{(1+\sqrt{x})^2}}{x^6 \sqrt{x^5}} dx.$

9. $\int \frac{\sqrt{1+\sqrt[3]{x^2}}}{x^2} dx.$

10. $\int \frac{\sqrt{1+x}}{x^2 \sqrt{x}} dx.$

11. $\int \frac{\sqrt[4]{(1+\sqrt{x})^3}}{x^8 \sqrt{x^7}} dx.$

12. $\int \frac{\sqrt[4]{(1+\sqrt[3]{x})^3}}{x^{12} \sqrt{x^7}} dx.$

13. $\int \frac{\sqrt[4]{(1+\sqrt[3]{x^2})^3}}{x^2 \sqrt[6]{x}} dx.$

14. $\int \frac{\sqrt{1+\sqrt[4]{x^3}}}{x^2 \sqrt[8]{x}} dx.$

15. $\int \frac{\sqrt[3]{1+\sqrt[4]{x^3}}}{x^2} dx.$

16. $\int \frac{\sqrt[3]{(1+\sqrt[4]{x^3})^2}}{x^2 \sqrt[4]{x}} dx.$

17. $\int \frac{\sqrt[5]{(1+\sqrt{x})^4}}{x^{10} \sqrt[9]{x}} dx.$

18. $\int \frac{\sqrt[5]{(1+\sqrt[3]{x})^4}}{x^5 \sqrt[3]{x^3}} dx.$

19. $\int \frac{\sqrt[5]{(1+\sqrt[3]{x^2})^4}}{x^2 \sqrt[5]{x}} dx.$

20. $\int \frac{\sqrt[5]{(1+\sqrt[4]{x^3})^4}}{x^{220} \sqrt[7]{x^7}} dx.$

21. $\int \frac{\sqrt[5]{1+\sqrt[5]{x^4}}}{x^{225} \sqrt[11]{x^{11}}} dx.$

22. $\int \frac{\sqrt{1+\sqrt[5]{x^4}}}{x^2 \sqrt[5]{x}} dx.$

23. $\int \frac{\sqrt[3]{1+\sqrt[5]{x^4}}}{x^2 \sqrt[15]{x}} dx.$

24. $\int \frac{\sqrt[3]{(1+\sqrt[5]{x^4})^2}}{x^2 \sqrt[3]{x}} dx.$

25. $\int \frac{\sqrt[4]{(1+\sqrt[5]{x^4})^3}}{x^2 \sqrt[5]{x^2}} dx.$

26. $\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{x^3 \sqrt{x}} dx.$

27. $\int \frac{\sqrt[3]{(1+\sqrt[4]{x})^2}}{x^{12} \sqrt[5]{x^5}} dx.$

28. $\int \frac{\sqrt[4]{1+\sqrt[3]{x}}}{x^{12} \sqrt[5]{x^5}} dx.$

29. $\int \frac{\sqrt[4]{1+\sqrt[3]{x^2}}}{x^6 \sqrt[5]{x^5}} dx.$

30. $\int \frac{\sqrt[3]{1+\sqrt[5]{x}}}{x^{15} \sqrt[4]{x^4}} dx.$

Задача 12. Вычислить площади фигур, ограниченных графиками функций.

1. $y = (x - 2)^3,$
 $y = 4x - 8.$

2. $y = x\sqrt{9 - x^2},$ $y = 0,$
 $(0 \leq x \leq 3).$

3. $y = 4 - x^2,$
 $y = x^2 - 2x.$

4. $y = \sin x \cos^2 x,$ $y = 0,$
 $(0 \leq x \leq \pi/2).$

5. $y = \sqrt{4 - x^2},$ $y = 0,$
 $x = 0, \quad x = 1.$

6. $y = x^2\sqrt{4 - x^2},$ $y = 0,$
 $(0 \leq x \leq 2).$

7. $y = \cos x \sin^2 x,$ $y = 0,$
 $(0 \leq x \leq \pi/2).$

8. $y = \sqrt{e^x - 1},$ $y = 0,$
 $x = \ln 2.$

9. $y = \frac{1}{x\sqrt{1 + \ln x}},$ $y = 0,$
 $x = 1, \quad x = e^3.$

10. $y = \arccos x,$ $y = 0,$
 $x = 0.$

11. $y = (x + 1)^2,$
 $y^2 = x + 1.$

12. $y = 2x - x^2 + 3,$
 $y = x^2 - 4x + 3.$

11. $y = x\sqrt{36 - x^2},$ $y = 0,$
 $(0 \leq x \leq 6).$

14. $x = \arccos y,$ $x = 0,$
 $y = 0.$

15. $y = \operatorname{arctg} x,$ $y = 0,$
 $x = \sqrt{3}.$

16. $y = x^2\sqrt{8 - x^2},$ $y = 0,$
 $(0 \leq x \leq 2\sqrt{2}).$

17. $x = \sqrt{e^y - 1},$ $x = 0,$
 $y = \ln 2.$

18. $y = x\sqrt{4 - x^2},$ $y = 0,$
 $(0 \leq x \leq 2).$

$$19. \begin{cases} y = \frac{x}{1+\sqrt{x}}, & y=0, \\ x = 1. \end{cases}$$

$$20. \begin{cases} y = \frac{1}{1+\cos x}, & y=0, \\ x = \pi/2, & x = -\pi/2. \end{cases}$$

$$21. \begin{cases} x = (y-2)^3, \\ x = 4y-8. \end{cases}$$

$$22. \begin{cases} y = \cos^5 x \sin 2x, & y=0, \\ (0 \leq x \leq \pi/2). \end{cases}$$

$$23. \begin{cases} y = \frac{x}{(x^2+1)^2}, & y=0, \\ x = 1. \end{cases}$$

$$24. \begin{cases} x = 4 - y^2, \\ x = y^2 - 2y. \end{cases}$$

$$25. \begin{cases} x = \frac{1}{y\sqrt{1+\ln y}}, & x=0, \\ y = 1, & y = e^3. \end{cases}$$

$$26. \begin{cases} y = \frac{e^{1/x}}{x^2}, & y=0, \\ x = 2, & x = 1. \end{cases}$$

$$27. \begin{cases} y = x^2 \sqrt{16-x^2}, & y=0, \\ (0 \leq x \leq 4). \end{cases}$$

$$28. \begin{cases} x = \sqrt{4-y^2}, & x=0, \\ y = 0, & y = 1. \end{cases}$$

$$29. \begin{cases} y = (x-1)^2, \\ y^2 = x-1. \end{cases}$$

$$30. \begin{cases} y = x^2 \cos x, & y=0, \\ (0 \leq x \leq \pi/2). \end{cases}$$

Задача 13. Вычислить площади фигур, ограниченных линиями, заданными уравнениями.

$$1. \begin{cases} x = 4\sqrt{2} \cos^3 t, \\ y = 2\sqrt{2} \sin^3 t, \\ x = 2 \quad (x \geq 2). \end{cases}$$

$$2. \begin{cases} x = \sqrt{2} \cos t, \\ y = 2\sqrt{2} \sin t, \\ y = 2 \quad (y \geq 2). \end{cases}$$

$$3. \begin{cases} x = 4(t - \sin t), \\ y = 4(1 - \cos t), \\ y = 4 \quad (0 < x < 8\pi, y \geq 4). \end{cases}$$

$$4. \begin{cases} x = 16 \cos^3 t, \\ y = 2 \sin^3 t, \\ x = 2 \quad (x \geq 2). \end{cases}$$

$$5. \begin{cases} x = 2 \cos t, \\ y = 6 \sin t, \\ y = 3 \quad (y \geq 3). \end{cases}$$

$$6. \begin{cases} x = 2(t - \sin t), \\ y = 2(1 - \cos t), \\ y = 3 \quad (0 < x < 4\pi, y \geq 3). \end{cases}$$

$$7. \begin{cases} x = 16 \cos^3 t, \\ y = \sin^3 t, \\ x = 6\sqrt{3} \quad (x \geq 6\sqrt{3}). \end{cases}$$

$$8. \begin{cases} x = 6 \cos t, \\ y = 2 \sin t, \\ y = \sqrt{3} \quad (y \geq \sqrt{3}). \end{cases}$$

$$9. \begin{cases} x = 3(t - \sin t), \\ y = 3(1 - \cos t), \\ y = 3 \quad (0 < x < 6\pi, y \geq 3). \end{cases}$$

10. $\begin{cases} x = 8\sqrt{2} \cos^3 t, \\ y = \sqrt{2} \sin^3 t, \\ x = 4 \quad (x \geq 4). \end{cases}$
11. $\begin{cases} x = 2\sqrt{2} \cos t, \\ y = 3\sqrt{2} \sin t, \\ y = 3 \quad (y \geq 3). \end{cases}$
12. $\begin{cases} x = 6(t - \sin t), \\ y = 6(1 - \cos t), \\ y = 9 \quad (0 < x < 12\pi, y \geq 9). \end{cases}$
13. $\begin{cases} x = 32 \cos^3 t, \\ y = \sin^3 t, \\ x = 4 \quad (x \geq 4). \end{cases}$
14. $\begin{cases} x = 3 \cos t, \\ y = 8 \sin t, \\ y = 4 \quad (y \geq 4). \end{cases}$
15. $\begin{cases} x = 32 \cos^3 t, \\ y = 3 \sin^3 t, \\ x = 12\sqrt{3} \quad (x \geq 12\sqrt{3}). \end{cases}$
16. $\begin{cases} x = 8 \cos^3 t, \\ y = 4 \sin^3 t, \\ x = 3\sqrt{3} \quad (x \geq 3\sqrt{3}). \end{cases}$
17. $\begin{cases} x = 6 \cos^3 t, \\ y = 4 \sin^3 t, \\ x = 2\sqrt{3} \quad (x \geq 2\sqrt{3}). \end{cases}$
18. $\begin{cases} x = 10(t - \sin t), \\ y = 10(1 - \cos t), \\ y = 15 \quad (0 < x < 20\pi, y \geq 15). \end{cases}$
19. $\begin{cases} x = 2\sqrt{2} \cos^3 t, \\ y = \sqrt{2} \sin^3 t, \\ x = 1 \quad (x \geq 1). \end{cases}$
20. $\begin{cases} x = \sqrt{2} \cos t, \\ y = 4\sqrt{2} \sin t, \\ y = 4 \quad (y \geq 4). \end{cases}$
21. $\begin{cases} x = t - \sin t, \\ y = 1 - \cos t, \\ y = 1 \quad (0 < x < 2\pi, y \geq 1). \end{cases}$
22. $\begin{cases} x = 8 \cos^3 t, \\ y = 8 \sin^3 t, \\ x = 1 \quad (x \geq 1). \end{cases}$
23. $\begin{cases} x = 9 \cos t, \\ y = 4 \sin t, \\ y = 2 \quad (y \geq 2). \end{cases}$
24. $\begin{cases} x = 8(t - \sin t), \\ y = 8(1 - \cos t), \\ y = 12 \quad (0 < x < 16\pi, y \geq 12). \end{cases}$
25. $\begin{cases} x = 24 \cos^3 t, \\ y = 2 \sin^3 t, \\ x = 9\sqrt{3} \quad (x \geq 9\sqrt{3}). \end{cases}$
26. $\begin{cases} x = 3 \cos t, \\ y = 8 \sin t, \\ y = 4\sqrt{3} \quad (y \geq 4\sqrt{3}). \end{cases}$
27. $\begin{cases} x = 2(t - \sin t), \\ y = 2(1 - \cos t), \\ y = 2 \quad (0 < x < 4\pi, y \geq 2). \end{cases}$
28. $\begin{cases} x = 4\sqrt{2} \cos^3 t, \\ y = \sqrt{2} \sin^3 t, \\ x = 2 \quad (x \geq 2). \end{cases}$
29. $\begin{cases} x = 2\sqrt{2} \cos t, \\ y = 5\sqrt{2} \sin t, \\ y = 5 \quad (y \geq 5). \end{cases}$
30. $\begin{cases} x = 4(t - \sin t), \\ y = 4(1 - \cos t), \\ y = 6 \quad (0 < x < 8\pi, y \geq 6). \end{cases}$

Задача 14. Вычислить площади фигур, ограниченных линиями, заданными в полярных координатах.

1. $r = 4 \cos 3\varphi, \quad r = 2 \quad (r \geq 2).$

2. $r = \cos 2\varphi.$

3. $r = \sqrt{3} \cos \varphi, \quad r = \sin \varphi,$
 $(0 \leq \varphi \leq \pi/2).$

4. $r = 4 \sin 3\varphi, \quad r = 2 \quad (r \geq 2).$

5. $r = 2 \cos \varphi, \quad r = 2\sqrt{3} \sin \varphi,$
 $(0 \leq \varphi \leq \pi/2).$

6. $r = \sin 3\varphi.$

7. $r = 6 \sin 3\varphi, \quad r = 3 \quad (r \geq 3).$

$r = \cos \varphi,$

9. $r = \sqrt{2} \sin(\varphi - \pi/4),$
 $(-\pi/4 \leq \varphi \leq \pi/2).$

11. $r = 6 \cos 3\varphi, \quad r = 3 \quad (r \geq 3).$

13. $r = \cos \varphi, \quad r = \sin \varphi,$
 $(0 \leq \varphi \leq \pi/2).$

15. $r = \cos \varphi, \quad r = 2 \cos \varphi.$

17. $r = 1 + \sqrt{2} \cos \varphi.$

19. $r = 1 + \sqrt{2} \sin \varphi.$

21. $r = (3/2) \cos \varphi, \quad r = (5/2) \cos \varphi.$

23. $r = \sin 6\varphi.$

25. $r = \cos \varphi + \sin \varphi.$

27. $r = 2 \cos 6\varphi.$

29. $r = 3 \sin \varphi, \quad r = 5 \sin \varphi.$

8. $r = \cos 3\varphi.$

$r = \sin \varphi,$

10. $r = \sqrt{2} \cos(\varphi - \pi/4),$
 $(0 \leq \varphi \leq 3\pi/4).$

12. $r = 1/2 + \sin \varphi.$

14. $r = 6 \sin \varphi, \quad r = 4 \sin \varphi.$

16. $r = \sin \varphi, \quad r = 2 \sin \varphi.$

18. $r = 1/2 + \cos \varphi.$

20. $r = (5/2) \sin \varphi, \quad r = (3/2) \sin \varphi.$

22. $r = 4 \cos 4\varphi.$

24. $r = 2 \cos \varphi, \quad r = 3 \cos \varphi.$

26. $r = 2 \sin 4\varphi.$

28. $r = \cos \varphi - \sin \varphi.$

30. $r = 2 \sin \varphi, \quad r = 4 \sin \varphi.$

Задача 15. Вычислить длины дуг кривых, заданных уравнениями в прямоугольной системе координат.

1. $y = \ln x, \quad \sqrt{3} \leq x \leq \sqrt{15}.$

2. $y = \frac{x^2}{4} - \frac{\ln x}{2}, \quad 1 \leq x \leq 2.$

3. $y = \sqrt{1-x^2} + \arcsin x, \quad 0 \leq x \leq 7/9.$

3. $y = \ln \frac{5}{2x}, \quad \sqrt{3} \leq x \leq \sqrt{8}.$

5. $y = -\ln \cos x, \quad 0 \leq x \leq \pi/6.$

6. $y = e^x + 6, \quad \ln \sqrt{8} \leq x \leq \ln \sqrt{15}.$

7. $y = 2 + \arcsin \sqrt{x} + \sqrt{x-x^2}, \quad 1/4 \leq x \leq 1.$

8. $y = \ln(x^2 - 1), \quad 2 \leq x \leq 3.$

$$9. y = \sqrt{1-x^2} + \arccos x, \quad 0 \leq x \leq 8/9.$$

$$11. y = 2 + \operatorname{ch} x, \quad 0 \leq x \leq 1.$$

$$13. y = e^x + 13, \quad \ln\sqrt{15} \leq x \leq \ln\sqrt{24}.$$

$$15. y = 2 - e^x, \quad \ln\sqrt{3} \leq x \leq \ln\sqrt{8}.$$

$$17. y = 1 - \ln \sin x, \quad \pi/3 \leq x \leq \pi/2.$$

$$19. y = \sqrt{x-x^2} - \arccos\sqrt{x} + 5, \quad 1/9 \leq x \leq 1.$$

$$21. y = \ln \sin x, \quad \pi/3 \leq x \leq \pi/2.$$

$$23. y = \operatorname{ch} x + 3, \quad 0 \leq x \leq 1.$$

$$25. y = \ln \cos x + 2, \quad 0 \leq x \leq \pi/6.$$

$$27. y = \frac{e^x + e^{-x}}{2} + 3, \quad 0 \leq x \leq 2.$$

$$29. y = \frac{e^x + e^{-x} + 3}{4}, \quad 0 \leq x \leq 2.$$

$$10. y = \ln(1-x^2), \quad 0 \leq x \leq 1/4.$$

$$12. y = 1 - \ln \cos x, \quad 0 \leq x \leq \pi/6.$$

$$14. y = -\arccos\sqrt{x} + \sqrt{x-x^2}, \quad 0 \leq x \leq 1/4.$$

$$16. y = \arcsin x - \sqrt{1-x^2}, \quad 0 \leq x \leq 15/16.$$

$$18. y = 1 - \ln(x^2 - 1), \quad 3 \leq x \leq 4.$$

$$20. y = -\arccos x + \sqrt{1-x^2} + 1, \quad 0 \leq x \leq 9/16.$$

$$22. y = \ln 7 - \ln x, \quad \sqrt{3} \leq x \leq \sqrt{8}.$$

$$24. y = 1 + \arcsin x - \sqrt{1-x^2}, \quad 0 \leq x \leq 3/4.$$

$$26. y = e^x + 26, \quad \ln\sqrt{8} \leq x \leq \ln\sqrt{24}.$$

$$28. y = \arccos\sqrt{x} - \sqrt{x-x^2} + 4, \quad 0 \leq x \leq 1/2.$$

$$30. y = e^x + e, \quad \ln\sqrt{3} \leq x \leq \ln\sqrt{15}.$$

Задача 16. Вычислить объемы тел, ограниченных поверхностями.

$$1. \frac{x^2}{9} + y^2 = 1, \quad z = y, \quad z = 0 \quad (y \geq 0).$$

$$2. z = x^2 + 4y^2, \quad z = 2.$$

$$3. \frac{x^2}{9} + \frac{y^2}{4} - z^2 = 1, \quad z = 0, \quad z = 3.$$

$$4. \frac{x^2}{9} + \frac{y^2}{4} - \frac{z^2}{36} = -1, \quad z = 12.$$

$$5. \frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{4} = 1, \quad z = 1, \quad z = 0.$$

$$6. x^2 + y^2 = 9, \quad z = y, \quad z = 0 \quad (y \geq 0).$$

$$7. z = x^2 + 9y^2, \quad z = 3.$$

$$8. \frac{x^2}{4} + y^2 - z^2 = 1, \quad z = 0, \quad z = 3.$$

$$9. \frac{x^2}{9} + \frac{y^2}{16} - \frac{z^2}{64} = -1, \quad z = 16.$$

$$10. \frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{16} = 1, \quad z = 2, \quad z = 0.$$

$$11. \frac{x^2}{3} + \frac{y^2}{4} = 1, \quad z = y\sqrt{3}, \quad z = 0 \quad (y \geq 0).$$

$$12. z = 2x^2 + 8y^2, \quad z = 4.$$

$$13. \frac{x^2}{81} + \frac{y^2}{25} - z^2 = 1, \quad z = 0, \quad z = 2.$$

$$14. \frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{36} = -1, \quad z = 12.$$

15. $\frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{36} = 1, z = 3, z = 0.$
16. $\frac{x^2}{3} + \frac{y^2}{16} = 1, z = y\sqrt{3}, z = 0 (y \geq 0).$
17. $z = x^2 + 5y^2, z = 5.$
18. $\frac{x^2}{9} + \frac{y^2}{4} - z^2 = 1, z = 0, z = 4.$
19. $\frac{x^2}{9} + \frac{y^2}{25} - \frac{z^2}{100} = -1, z = 20.$
16. $\frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{64} = 1, z = 4, z = 0.$
21. $\frac{x^2}{27} + \frac{y^2}{25} = 1, z = \frac{y}{\sqrt{3}}, z = 0 (y \geq 0).$
22. $z = 4x^2 + 9y^2, z = 6.$
23. $x^2 + \frac{y^2}{4} - z^2 = 1, z = 0, z = 3.$
24. $\frac{x^2}{25} + \frac{y^2}{9} - \frac{z^2}{100} = -1, z = 20.$
25. $\frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{100} = 1, z = 5, z = 0.$
26. $\frac{x^2}{27} + y^2 = 1, z = \frac{y}{\sqrt{3}}, z = 0 (y \geq 0).$
27. $z = 2x^2 + 18y^2, z = 6.$
28. $\frac{x^2}{25} + \frac{y^2}{9} - z^2 = 1, z = 0, z = 2.$
29. $\frac{x^2}{16} + \frac{y^2}{9} - \frac{z^2}{64} = -1, z = 16.$
30. $\frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{144} = 1, z = 6, z = 0.$

Задача 17. Вычислить объемы тел, образованных вращением фигур, ограниченных графиками функций. В вариантах 1–16 ось вращения Ox , в вариантах 17–30 ось вращения Oy .

1. $y = -x^2 + 5x - 6, y = 0.$
2. $2x - x^2 - y = 0, 2x^2 - 4x + y = 0.$
3. $y = 3\sin x, y = \sin x, 0 \leq x \leq \pi.$
4. $y = 5\cos x, y = \cos x, x = 0, x \geq 0.$
5. $y = \sin^2 x, x = \pi/2, y = 0.$
6. $x = \sqrt[3]{y-2}, x = 1, y = 1.$
7. $y = xe^x, y = 0, x = 1.$
8. $y = 2x - x^2, y = -x + 2, x = 0.$
9. $y = 2x - x^2, y = -x + 2.$
10. $y = e^{1-x}, y = 0, x = 0, x = 1.$
11. $y = x^2, y^2 - x = 0.$
12. $x^2 + (y-2)^2 = 1.$
13. $y = 1 - x^2, x = 0, x = \sqrt{y-1}, x = 1.$
14. $y = x^2, y = 1, x = 2.$
15. $y = x^2, y = \sqrt{x}.$
16. $y = \sin(\pi x/2), y = x^2.$
17. $y = \arccos(x/3), y = \arccos x, y = 0.$
18. $y = \arcsin(x/5), y = \arcsin x, y = \pi/2.$
19. $y = x^2, x = 2, y = 0.$
20. $y = x^2 + 1, y = x, x = 0, y = 0.$

$$21. y = \sqrt{x-1}, \quad y=0, \quad y=1, \quad x=0,5.$$

$$23. y = (x-1)^2, \quad y=1.$$

$$25. y = x^3, \quad y = x^2.$$

$$27. y = \arcsin x, \quad y = \arccos x, \quad y=0.$$

$$29. y = x^3, \quad y = x.$$

$$22. y = \ln x, \quad x=2, \quad y=0.$$

$$24. y^2 = x-2, \quad y=0, \quad y = x^3, \quad y=1.$$

$$26. y = \arccos(x/5), \quad y = \arccos(x/3), \quad y=0.$$

$$28. y = x^2 - 2x + 1, \quad x=2, \quad y=0.$$

$$30. y = \arccos x, \quad y = \arcsin x, \quad x=0.$$

Задача 18. Изменить порядок интегрирования.

$$1. \int_{-2}^{-1} dy \int_{-\sqrt{2+y}}^0 f dx + \int_{-1}^0 dy \int_{-\sqrt{-y}}^0 f dx.$$

$$2. \int_0^1 dy \int_{-\sqrt{y}}^0 f dx + \int_1^{\sqrt{2}} dy \int_{-\sqrt{-y}}^0 f dx.$$

$$3. \int_0^1 dy \int_0^y f dx + \int_1^{\sqrt{2}} dy \int_0^{\sqrt{2-y^2}} f dx.$$

$$4. \int_0^1 dy \int_0^{\sqrt{y}} f dx + \int_1^2 dy \int_0^{\sqrt{2-y}} f dx.$$

$$5. \int_{-\sqrt{2}}^{-1} dx \int_{-\sqrt{2-x^2}}^0 f dy + \int_{-1}^0 dx \int_x^0 f dy.$$

$$6. \int_0^{1/\sqrt{2}} dy \int_0^{\arcsin y} f dx + \int_{1/\sqrt{2}}^1 dy \int_0^{\arccos y} f dx.$$

$$7. \int_{-2}^{-1} dy \int_0^{\sqrt{2+y}} f dx + \int_{-1}^0 dy \int_0^{\sqrt{-y}} f dx.$$

$$8. \int_0^1 dy \int_{-\sqrt{y}}^0 f dx + \int_1^e dy \int_{-1}^{-\ln y} f dx.$$

$$9. \int_{-\sqrt{2}}^{-1} dx \int_0^{\sqrt{2-x^2}} f dy + \int_{-1}^0 dx \int_0^{x^2} f dy.$$

$$10. \int_{-2}^{-\sqrt{3}} dx \int_{-\sqrt{4-x^2}}^0 f dy + \int_{-\sqrt{3}}^0 dx \int_{\sqrt{4-x^2}-2}^0 f dy.$$

$$11. \int_0^1 dx \int_{1-x^2}^1 f dy + \int_1^e dx \int_{\ln x}^1 f dy.$$

$$12. \int_0^1 dy \int_0^{\sqrt[3]{y}} f dx + \int_1^2 dy \int_0^{2-y} f dx.$$

$$13. \int_0^{\pi/4} dy \int_0^{\sin y} f dx + \int_{\pi/4}^{\pi/2} dy \int_0^{\cos y} f dx.$$

$$14. \int_{-2}^{-1} dx \int_{-(2+x)}^0 f dy + \int_{-1}^0 dx \int_{\sqrt[3]{x}}^0 f dy.$$

$$15. \int_0^1 dy \int_0^{\sqrt{y}} f dx + \int_1^e dy \int_{\ln y}^1 f dx.$$

$$16. \int_0^1 dy \int_{-\sqrt{y}}^0 f dx + \int_1^2 dy \int_{-\sqrt{2-y}}^0 f dx.$$

$$17. \int_0^1 dy \int_{-y}^0 f dx + \int_1^{\sqrt{2}} dy \int_{-\sqrt{2-y^2}}^0 f dx.$$

$$18. \int_0^1 dy \int_0^{y^2} f dx + \int_1^2 dy \int_0^{2-y} f dx.$$

$$19. \int_0^{\sqrt{3}} dx \int_{\sqrt{4-x^2}-2}^0 f dy + \int_{\sqrt{3}}^2 dx \int_{-\sqrt{4-x^2}}^0 f dy .$$

$$20. \int_{-2}^{-1} dy \int_{-(2+y)}^0 f dx + \int_{-1}^0 dy \int_{\sqrt[3]{y}}^0 f dx .$$

$$21. \int_0^1 dy \int_0^y f dx + \int_1^e dy \int_{\ln y}^1 f dx .$$

$$22. \int_0^1 dx \int_0^{x^2} f dy + \int_1^{\sqrt{2}} dx \int_0^{\sqrt{2-x^2}} f dy .$$

$$23. \int_0^{\pi/4} dx \int_0^{\sin x} f dy + \int_{\pi/4}^{\pi/2} dx \int_0^{\cos x} f dy .$$

$$24. \int_{-\sqrt{2}}^{-1} dy \int_{-\sqrt{2-y^2}}^0 f dx + \int_{-1}^0 dy \int_y^0 f dx .$$

$$25. \int_0^1 dx \int_0^{x^2} f dy + \int_1^2 dx \int_0^{2-x} f dy .$$

$$26. \int_0^{\sqrt{3}} dx \int_0^{2-\sqrt{4-x^2}} f dy + \int_{\sqrt{3}}^2 dx \int_0^{\sqrt{4-x^2}} f dy .$$

$$27. \int_0^1 dx \int_{-\sqrt{x}}^0 f dy + \int_1^2 dx \int_{-\sqrt{2-x}}^0 f dy .$$

$$28. \int_0^1 dx \int_0^x f dy + \int_1^{\sqrt{2}} dx \int_0^{\sqrt{2-x^2}} f dy .$$

$$29. \int_0^1 dy \int_0^{\sqrt{y}} f dx + \int_1^{\sqrt{2}} dy \int_0^{\sqrt{2-y^2}} f dx .$$

$$30. \int_0^1 dx \int_0^{\sqrt{x}} f dy + \int_1^2 dx \int_0^{\sqrt{2-x}} f dy .$$

Задача 19. Вычислить.

$$1. \iint_D (12x^2y^2 + 16x^3y^3) dx dy;$$

D: $x = 1, y = x^2, y = -\sqrt{x}$.

$$2. \iint_D (9x^2y^2 + 48x^3y^3) dx dy;$$

D: $x = 1, y = \sqrt{x}, y = -x^2$.

$$3. \iint_D (36x^2y^2 - 96x^3y^3) dx dy;$$

D: $x = 1, y = \sqrt[3]{x}, y = -x^3$.

$$4. \iint_D (18x^2y^2 + 32x^3y^3) dx dy;$$

D: $x = 1, y = x^3, y = -\sqrt[3]{x}$.

$$5. \iint_D (27x^2y^2 + 48x^3y^3) dx dy;$$

D: $x = 1, y = x^2, y = -\sqrt[3]{x}$.

$$6. \iint_D (18x^2y^2 + 32x^3y^3) dx dy;$$

D: $x = 1, y = \sqrt[3]{x}, y = -x^2$.

$$7. \iint_D (18x^2y^2 + 32x^3y^3) dx dy;$$

D: $x = 1, y = x^3, y = -\sqrt{x}$.

$$8. \iint_D (27x^2y^2 + 48x^3y^3) dx dy;$$

D: $x = 1, y = \sqrt{x}, y = -x^3$.

$$9. \iint_D (4xy + 3x^2y^2) dx dy;$$

D: $x = 1, y = x^2, y = -\sqrt{x}$.

$$10. \iint_D (12xy + 9x^2y^2) dx dy;$$

D: $x = 1, y = \sqrt{x}, y = -x^2$.

11. $\iint_D (8xy + 9x^2y^2) dx dy;$
D: $x = 1, y = \sqrt[3]{x}, y = -x^3.$

13. $\iint_D (12xy + 27x^2y^2) dx dy;$
D: $x = 1, y = x^2, y = -\sqrt[3]{x}.$

15. $\iint_D \left(\frac{4}{5}xy + \frac{9}{11}x^2y^2 \right) dx dy;$
D: $x = 1, y = x^3, y = -\sqrt{x}.$

17. $\iint_D (24xy - 48x^3y^3) dx dy;$
D: $x = 1, y = x^2, y = -\sqrt{x}.$

19. $\iint_D (4xy + 16x^3y^3) dx dy;$
D: $x = 1, y = \sqrt[3]{x}, y = -x^3.$

21. $\iint_D (44xy + 16x^3y^3) dx dy;$
D: $x = 1, y = x^2, y = -\sqrt[3]{x}.$

23. $\iint_D (xy - 4x^3y^3) dx dy;$
D: $x = 1, y = x^3, y = -\sqrt{x}.$

25. $\iint_D \left(6x^2y^2 + \frac{25}{3}x^4y^4 \right) dx dy;$
D: $x = 1, y = x^2, y = -\sqrt{x}.$

27. $\iint_D \left(3x^2y^2 + \frac{50}{3}x^4y^4 \right) dx dy;$
D: $x = 1, y = \sqrt[3]{x}, y = -x^3.$

2.29. $\iint_D (54x^2y^2 + 150x^4y^4) dx dy;$
D: $x = 1, y = x^2, y = -\sqrt[3]{x}.$

12. $\iint_D (24xy + 18x^2y^2) dx dy;$
D: $x = 1, y = x^3, y = -\sqrt[3]{x}.$

14. $\iint_D (8xy + 18x^2y^2) dx dy;$
D: $x = 1, y = \sqrt[3]{x}, y = -x^2.$

16. $\iint_D \left(\frac{4}{5}xy + 9x^2y^2 \right) dx dy;$
D: $x = 1, y = \sqrt{x}, y = -x^3.$

18. $\iint_D (6xy + 24x^3y^3) dx dy;$
D: $x = 1, y = \sqrt{x}, y = -x^2.$

20. $\iint_D (4xy + 16x^3y^3) dx dy;$
D: $x = 1, y = x^3, y = -\sqrt[3]{x}.$

22. $\iint_D (4xy + 176x^3y^3) dx dy;$
D: $x = 1, y = \sqrt[3]{x}, y = -x^3.$

24. $\iint_D (4xy + 176x^3y^3) dx dy;$
D: $x = 1, y = \sqrt{x}, y = -x^3.$

26. $\iint_D (9x^2y^2 + 25x^4y^4) dx dy;$
D: $x = 1, y = \sqrt{x}, y = -x^2.$

28. $\iint_D (9x^2y^2 + 25x^4y^4) dx dy;$
D: $x = 1, y = x^3, y = -\sqrt[3]{x}.$

2.30. $\iint_D (xy - 9x^5y^5) dx dy;$
D: $x = 1, y = \sqrt[3]{x}, y = -x^2.$

Задача 20. Вычислить.

1. $\iint_D ye^{xy/2} dx dy;$

D: $y = \ln 2, y = \ln 3, x = 2, x = 4.$

3. $\iint_D y \cos xy dx dy;$

D: $y = \pi/2, y = \pi, x = 1, x = 2.$

5. $\iint_D y \sin xy dx dy;$

D: $y = \pi/2, y = \pi, x = 1, x = 2.$

7. $\iint_D 4ye^{2xy} dx dy;$

D: $y = \ln 3, y = \ln 4, x = \frac{1}{2}, x = 1.$

9. $\iint_D y \cos 2xy dx dy;$

D: $y = \frac{\pi}{2}, y = \pi, x = \frac{1}{2}, x = 1.$

11. $\iint_D 12y \sin 2xy dx dy;$

D: $y = \frac{\pi}{4}, y = \frac{\pi}{2}, x = 2, x = 3.$

13. $\iint_D ye^{xy/4} dx dy;$

D: $y = \ln 2, y = \ln 3, x = 4, x = 8.$

15. $\iint_D 2y \cos 2xy dx dy;$

D: $y = \frac{\pi}{4}, y = \frac{\pi}{2}, x = 1, x = 2.$

17. $\iint_D y \sin xy dx dy;$

D: $y = \pi, y = 2\pi, x = \frac{1}{2}, x = 1.$

2. $\iint_D y^2 \sin \frac{xy}{2} dx dy;$

D: $x = 0, y = \sqrt{\pi}, y = \frac{x}{2}.$

4. $\iint_D y^2 e^{-xy/4} dx dy;$

D: $x = 0, y = 2, y = x.$

6. $\iint_D y^2 \cos \frac{xy}{2} dx dy;$

D: $x = 0, y = \sqrt{\pi/2}, y = x/2.$

8. $\iint_D 4y^2 \sin xy dx dy;$

D: $x = 0, y = \sqrt{\frac{\pi}{2}}, y = x.$

10. $\iint_D y^2 e^{-xy/8} dx dy;$

D: $x = 0, y = 2, y = \frac{x}{2}.$

12. $\iint_D y^2 \cos xy dx dy;$

D: $x = 0, y = \sqrt{\pi}, y = x.$

14. $\iint_D y^2 \sin 2xy dx dy;$

D: $x = 0, y = \sqrt{2\pi}, y = 2x.$

16. $\iint_D y^2 e^{-xy/2} dx dy;$

D: $x = 0, y = \sqrt{2}, y = x.$

18. $\iint_D y^2 \cos 2xy dx dy;$

D: $x = 0, y = \sqrt{\frac{\pi}{2}}, y = \frac{x}{2}.$

$$19. \iint_D 8ye^{4xy} dx dy;$$

$$D: y = \ln 3, y = \ln 4, x = \frac{1}{4}, x = \frac{1}{2}.$$

$$20. \iint_D 3y^2 \sin \frac{xy}{2} dx dy;$$

$$D: x = 0, y = \sqrt{\frac{4\pi}{3}}, y = \frac{2}{3}x.$$

$$21. \iint_D y \cos xy dx dy;$$

$$D: y = \pi, y = 3\pi, x = 1/2, x = 1.$$

$$22. \iint_D y^2 e^{-xy/2} dx dy;$$

$$D: x = 0, y = 1, y = \frac{x}{2}.$$

$$23. \iint_D y \sin 2xy dx dy;$$

$$D: y = \pi/2, y = 3\pi/2, x = 1/2, x = 2.$$

$$24. \iint_D y^2 \cos xy dx dy;$$

$$D: x = 0, y = \sqrt{\pi}, y = 2x.$$

$$25. \iint_D 6ye^{xy/3} dx dy;$$

$$D: y = \ln 2, y = \ln 3, x = 3, x = 6.$$

$$26. \iint_D y^2 \sin \frac{xy}{2} dx dy;$$

$$D: x = 0, y = \sqrt{\pi}, y = x.$$

$$27. \iint_D y \cos 2xy dx dy;$$

$$D: y = \pi/2, y = 3\pi/2, x = 1/2, x = 2.$$

$$28. \iint_D y^2 e^{-xy/8} dx dy;$$

$$D: x = 0, y = 4, y = 2x.$$

$$29. \iint_D 3y \sin xy dx dy;$$

$$D: y = \pi/2, y = 3\pi, x = 1, x = 3.$$

$$30. \iint_D y^2 \cos \frac{xy}{2} dx dy;$$

$$D: x = 0, y = \sqrt{2\pi}, y = 2x.$$

Задача 21. Найти площадь фигуры, ограниченной данными линиями, используя двойной интеграл.

$$1. y = 3/x, y = 4e^x, y = 3, y = 4.$$

$$2. x = \sqrt{36 - y^2}, x = 6 - \sqrt{36 - y^2}.$$

$$3. x^2 + y^2 = 72, 6y = -x^2 (y \leq 0).$$

$$4. x = 8 - y^2, x = -2y.$$

$$5. y = \frac{3}{x}, y = 8e^x, y = 3, y = 8.$$

$$6. y = \frac{\sqrt{x}}{2}, y = \frac{1}{2x}, x = 16.$$

$$7. x = 5 - y^2, x = -4y.$$

$$8. x^2 + y^2 = 12, -\sqrt{6}y = x^2 (y \leq 0).$$

$$9. x^2 + y^2 = 12, x\sqrt{6} = y^2 (x \geq 0).$$

$$10. y = \frac{3}{2}\sqrt{x}, y = \frac{3}{2x}, x = 9.$$

$$11. y = \sqrt{24 - x^2}, 2\sqrt{3}y = x^2, x = 0 \ (x \geq 0).$$

$$13. y = 20 - x^2, y = -8x.$$

$$15. y = 32 - x^2, y = -4x.$$

$$17. x^2 + y^2 = 36, 3\sqrt{2}y = x^2 \ (y \geq 0).$$

$$19. y = 6 - \sqrt{36 - x^2}, y = \sqrt{36 - x^2}, x = 0 \ (x \geq 0).$$

$$21. y = \sqrt{x}, y = 1/x, x = 16.$$

$$23. x = 27 - y^2, x = -6y.$$

$$25. y = \sqrt{6 - x^2}, y = \sqrt{6} - \sqrt{6 - x^2}.$$

$$27. y = \sin x, y = \cos x, x = 0, \ (x \leq 0).$$

$$29. y = 3\sqrt{x}, y = 3/x, x = 9.$$

$$12. y = \sin x, y = \cos x, x = 0, \ (x \geq 0).$$

$$14. y = \sqrt{18 - x^2}, y = 3\sqrt{2} - \sqrt{18 - x^2}.$$

$$16. y = 2/x, y = 5e^x, y = 2, y = 5.$$

$$18. y = 3\sqrt{x}, y = 3/x, x = 4.$$

$$20. y = 25/4 - x^2, y = x - 5/2.$$

$$22. y = 2/x, y = 7e^x, y = 2, y = 7.$$

$$24. x = \sqrt{72 - y^2}, 6x = y^2, y = 0 \ (y \geq 0).$$

$$26. y = \frac{3}{2}\sqrt{x}, y = \frac{3}{2x}, x = 4.$$

$$28. y = \frac{1}{x}, y = 6e^x, y = 1, y = 6.$$

$$30. y = 11 - x^2, y = -10x.$$

Задача 22. Найти объем тела, заданного ограничивающими его поверхностями

(использовать двойной интеграл).

$$1. \begin{cases} y = 16\sqrt{2x}, & y = \sqrt{2x}, \\ z = 0, & x + z = 2. \end{cases}$$

$$2. \begin{cases} y = 5\sqrt{x}, & y = 5x/3, \\ z = 0, & z = 5 + 5\sqrt{x}/3. \end{cases}$$

$$3. \begin{cases} x^2 + y^2 = 2, & y = \sqrt{x}, & y = 0, \\ z = 0, & z = 15x. \end{cases}$$

$$4. \begin{cases} x + y = 2, & y = \sqrt{x}, \\ z = 12y, & z = 0. \end{cases}$$

$$5. \begin{cases} x = 20\sqrt{2y}, & x = 5\sqrt{2y}, \\ z = 0, & z + y = 1/2. \end{cases}$$

$$6. \begin{cases} x = 5\sqrt{y}/2, & x = 5y/6, \\ z = 0, & z = \frac{5}{6}(3 + \sqrt{y}). \end{cases}$$

$$7. \begin{cases} x^2 + y^2 = 2, & x = \sqrt{y}, & x = 0, \\ z = 0, & z = 30y. \end{cases}$$

$$8. \begin{cases} x + y = 2, & x = \sqrt{y}, \\ z = 12x/5, & z = 0. \end{cases}$$

$$9. \begin{cases} y = 17\sqrt{2x}, & y = 2\sqrt{2x}, \\ z = 0, & x + z = 1/2. \end{cases}$$

$$10. \begin{cases} y = 5\sqrt{x}/3, & y = 5x/9, \\ z = 0, & z = 5(3 + \sqrt{x})/9. \end{cases}$$

$$11. \begin{cases} x^2 + y^2 = 8, & y = \sqrt{2x}, & y = 0, \\ z = 0, & z = 15x/11. \end{cases}$$

$$12. \begin{cases} x + y = 4, & y = \sqrt{2x}, \\ z = 3y, & z = 0. \end{cases}$$

$$13. \begin{cases} x = \frac{5}{6}\sqrt{y}, & x = \frac{5}{18}y, \\ z = 0, & z = \frac{5}{18}(3 + \sqrt{y}). \end{cases}$$

$$14. \begin{cases} x = 19\sqrt{2y}, & x = 4\sqrt{2y}, \\ z = 0, & z + y = 2. \end{cases}$$

$$15. \begin{cases} x^2 + y^2 = 8, & x = \sqrt{2y}, & x = 0, \\ z = 30y/11, & z = 0. \end{cases}$$

$$16. \begin{cases} x + y = 4, & x = \sqrt{2y}, \\ z = 3x/5, & z = 0. \end{cases}$$

$$17. \begin{cases} y = 6\sqrt{3x}, & y = \sqrt{3x}, \\ z = 0, & x + z = 3. \end{cases}$$

$$18. \begin{cases} y = \frac{5}{6}\sqrt{x}, & y = \frac{5}{18}x, \\ z = 0, & z = \frac{5}{18}(3 + \sqrt{x}). \end{cases}$$

$$19. \begin{cases} x^2 + y^2 = 18, & y = \sqrt{3x}, & y = 0, \\ z = 0, & z = 5x/11. \end{cases}$$

$$20. \begin{cases} x + y = 6, & y = \sqrt{3x}, \\ z = 4y, & z = 0. \end{cases}$$

$$21. \begin{cases} x = 7\sqrt{3y}, & x = 2\sqrt{3y}, \\ z = 0, & z + y = 3. \end{cases}$$

$$22. \begin{cases} x = 5\sqrt{y}/3, & x = 5y/9, \\ z = 0, & z = 5(3 + \sqrt{y})/9. \end{cases}$$

$$23. \begin{cases} x^2 + y^2 = 18, & x = \sqrt{3y}, & x = 0, \\ z = 0, & z = 10y/11. \end{cases}$$

$$24. \begin{cases} x + y = 6, & x = \sqrt{3y}, \\ z = 4x/5, & z = 0. \end{cases}$$

$$25. \begin{cases} y = \sqrt{15x}, & y = \sqrt{15x}, \\ z = 0, & z = \sqrt{15}(1 + \sqrt{x}). \end{cases}$$

$$26. \begin{cases} x^2 + y^2 = 50, & y = \sqrt{5x}, \\ y = 0, & z = 0, & z = 3x/11. \end{cases}$$

$$27. \begin{cases} x + y = 8, & y = \sqrt{4x}, \\ z = 3y, & z = 0. \end{cases}$$

$$28. \begin{cases} x = 16\sqrt{2y}, & x = \sqrt{2y}, \\ z + y = 2, & z = 0. \end{cases}$$

$$29. \begin{cases} x = \sqrt{y}, & x = 15y, \\ z = 0, & z = 15(1 + \sqrt{y}). \end{cases}$$

$$30. \begin{cases} x^2 + y^2 = 50, & x = \sqrt{5y}, \\ x = 0, & z = 0, & z = 6y/11. \end{cases}$$

Задача 23. Найти объем тела, заданного ограничивающими его поверхностями

(использовать двойной интеграл).

1. $x^2 + y^2 = 2y,$
 $z = 5/4 - x^2, z = 0.$

$x^2 + y^2 = 8\sqrt{2}x,$
3. $z = x^2 + y^2 - 64,$
 $z = 0 (z \geq 0).$

$x^2 + y^2 = 6x, x^2 + y^2 = 9x,$
5. $z = \sqrt{x^2 + y^2}, z = 0,$
 $y = 0 (y \leq 0)$

7. $x^2 + y^2 = 2y,$
 $z = 9/4 - x^2, z = 0.$

$x^2 + y^2 + 2\sqrt{2}y = 0,$
9. $z = x^2 + y^2 - 4,$
 $z = 0 (z \geq 0).$

$x^2 + y^2 = 7x, x^2 + y^2 = 9x,$
11. $z = \sqrt{x^2 + y^2}, z = 0,$
 $y = 0 (y \leq 0)$

13. $x^2 + y^2 = 2y,$
 $z = 13/4 - x^2, z = 0.$

$x^2 + y^2 = 6\sqrt{2}x,$
15. $z = x^2 + y^2 - 36,$
 $z = 0 (z \geq 0).$

17. $x^2 + y^2 = 4x,$
 $z = 12 - y^2, z = 0.$

2. $x^2 + y^2 = y, x^2 + y^2 = 4y,$
 $z = \sqrt{x^2 + y^2}, z = 0.$

4. $x^2 + y^2 + 4x = 0,$
 $z = 8 - y^2, z = 0.$

$x^2 + y^2 = 6\sqrt{2}y,$
6. $z = x^2 + y^2 - 36,$
 $z = 0 (z \geq 0).$

8. $x^2 + y^2 = 2y, x^2 + y^2 = 5y,$
 $z = \sqrt{x^2 + y^2}, z = 0.$

10. $x^2 + y^2 = 4x,$
 $z = 10 - y^2, z = 0.$

$x^2 + y^2 = 8\sqrt{2}y,$
12. $z = x^2 + y^2 - 64,$
 $z = 0 (z \geq 0).$

14. $x^2 + y^2 = 3y, x^2 + y^2 = 6y,$
 $z = \sqrt{x^2 + y^2}, z = 0.$

$x^2 + y^2 = 2\sqrt{2}y,$
16. $z = x^2 + y^2 - 4,$
 $z = 0 (z \geq 0).$

$x^2 + y^2 = 8x, x^2 + y^2 = 11x,$
18. $z = \sqrt{x^2 + y^2}, z = 0,$
 $y = 0 (y \leq 0)$

$$\begin{aligned}x^2 + y^2 &= 4\sqrt{2}x, \\19. \quad z &= x^2 + y^2 - 16, \\z &= 0 \quad (z \geq 0).\end{aligned}$$

$$\begin{aligned}21. \quad x^2 + y^2 &= 4y, \quad x^2 + y^2 = 7y, \\z &= \sqrt{x^2 + y^2}, \quad z = 0.\end{aligned}$$

$$\begin{aligned}23. \quad x^2 + y^2 + 2x &= 0, \\z &= 17/4 - y^2, \quad z = 0.\end{aligned}$$

$$\begin{aligned}25. \quad x^2 + y^2 + 2\sqrt{2}x &= 0, \\z &= x^2 + y^2 - 4, \\z &= 0 \quad (z \geq 0).\end{aligned}$$

$$\begin{aligned}27. \quad x^2 + y^2 &= 10x, \quad x^2 + y^2 = 13x, \\z &= \sqrt{x^2 + y^2}, \quad z = 0, \\y &= 0 \quad (y \geq 0)\end{aligned}$$

$$\begin{aligned}29. \quad x^2 + y^2 &= 2x, \\z &= 21/4 - y^2, \quad z = 0.\end{aligned}$$

$$\begin{aligned}20. \quad x^2 + y^2 &= 4y, \\z &= 4 - x^2, \quad z = 0.\end{aligned}$$

$$\begin{aligned}22. \quad x^2 + y^2 &= 4\sqrt{2}y, \\z &= x^2 + y^2 - 16, \\z &= 0 \quad (z \geq 0).\end{aligned}$$

$$\begin{aligned}24. \quad x^2 + y^2 &= 9x, \quad x^2 + y^2 = 12x, \\z &= \sqrt{x^2 + y^2}, \quad z = 0, \\y &= 0 \quad (y \geq 0)\end{aligned}$$

$$\begin{aligned}26. \quad x^2 + y^2 &= 4y, \\z &= 6 - x^2, \quad z = 0.\end{aligned}$$

$$\begin{aligned}28. \quad x^2 + y^2 &= 2\sqrt{2}x, \\z &= x^2 + y^2 - 4, \\z &= 0 \quad (z \geq 0).\end{aligned}$$

$$\begin{aligned}30. \quad x^2 + y^2 &= 5y, \quad x^2 + y^2 = 8y, \\z &= \sqrt{x^2 + y^2}, \quad z = 0.\end{aligned}$$