

СБОРНИК №3

ЗАДАНИЙ

ПО ВЫСШЕЙ МАТЕМАТИКЕ

для студентов инженерных специальностей

2007

ДИФФЕРЕНЦИАЛЬНЫЕ УРАВНЕНИЯ И РЯДЫ

В сборнике подобраны однотипные одинаковой степени сложности задачи по темам. Каждое задание содержит 30 вариантов. Предлагаемые в сборнике задачи предназначены для студентов дневной и заочной форм обучения. Сборник содержит задания по темам, соответствующим программе по курсу высшей математики для студентов инженерных специальностей дневной и заочной формы обучения полного и сокращенного курса.

В сборник включены следующие темы : дифференциальные уравнения первого порядка, второго, числовые ряды, степенные ряды, приложения рядов.

Задача 1. Найти общее решение дифференциального уравнения.

1. $4x dx - 3y dy = 3x^2 y dy - 2xy^2 dx.$

2. $x\sqrt{1+y^2} + yy'\sqrt{1+x^2} = 0.$

3. $\sqrt{4+y^2} dx - y dy = x^2 y dy.$

4. $\sqrt{3+y^2} dx - y dy = x^2 y dy.$

5. $6x dx - 6y dy = 2x^2 y dy - 3xy^2 dx.$

6. $x\sqrt{3+y^2} dx + y\sqrt{2+x^2} dy = 0.$

7. $(e^{2x} + 5) dy + y e^{2x} dx = 0.$

8. $y' y \sqrt{\frac{1-x^2}{1-y^2}} + 1 = 0.$

9. $6x dx - 6y dy = 3x^2 y dy - 2xy^2 dx.$

10. $x\sqrt{5+y^2} dx + y\sqrt{4+x^2} dy = 0.$

11. $y(4 + e^x) dy - e^x dx = 0.$

12. $\sqrt{4-x^2} y' + xy^2 + x = 0.$

13. $2x dx - 2y dy = x^2 y dy - 2xy^2 dx.$

14. $x\sqrt{4+y^2} dx + y\sqrt{1+x^2} dy = 0.$

15. $(e^x + 8) dy - y e^x dx = 0.$

16. $\sqrt{5+y^2} + y' y \sqrt{1-x^2} = 0.$

17. $6x dx - y dy = yx^2 dy - 3xy^2 dx.$

18. $y \ln y + xy' = 0.$

19. $(1 + e^x) y' = y e^x.$

20. $\sqrt{1-x^2} y' + xy^2 + x = 0.$

21. $6x dx - 2y dy = 2yx^2 dy - 3xy^2 dx.$

22. $y(1 + \ln y) + xy' = 0.$

23. $(3 + e^x) yy' = e^x.$

24. $\sqrt{3+y^2} + \sqrt{1-x^2} yy' = 0.$

25. $x dx - y dy = yx^2 dy - xy^2 dx.$

26. $\sqrt{5+y^2} dx + 4(x^2 y + y) dy = 0.$

27. $(1+e^x) yy' = e^x.$

28. $3(x^2 y + y) dy + \sqrt{2+y^2} dx = 0.$

29. $2x dx - y dy = yx^2 dy - xy^2 dx.$

30. $2x + 2xy^2 + \sqrt{2-x^2} y' = 0.$

Задача 2. Найти общее решение дифференциального уравнения.

1. $y' = \frac{y^2}{x^2} + 4\frac{y}{x} + 2.$

2. $xy' = \frac{3y^3 + 2yx^2}{2y^2 + x^2}.$

3. $y' = \frac{x+y}{x-y}.$

4. $xy' = \sqrt{x^2 + y^2} + y.$

5. $2y' = \frac{y^2}{x^2} + 6\frac{y}{x} + 3.$

6. $xy' = \frac{3y^3 + 4yx^2}{2y^2 + 2x^2}.$

7. $y' = \frac{x+2y}{2x-y}.$

8. $xy' = 2\sqrt{x^2 + y^2} + y.$

9. $3y' = \frac{y^2}{x^2} + 8\frac{y}{x} + 4.$

10. $xy' = \frac{3y^3 + 6yx^2}{2y^2 + 3x^2}.$

11. $y' = \frac{x^2 + xy - y^2}{x^2 - 2xy}.$

12. $xy' = \sqrt{2x^2 + y^2} + y.$

13. $y' = \frac{y^2}{x^2} + 6\frac{y}{x} + 6.$

14. $xy' = \frac{3y^3 + 8yx^2}{2y^2 + 4x^2}.$

15. $y' = \frac{x^2 + 2xy - y^2}{2x^2 - 2xy}.$

16. $xy' = 3\sqrt{x^2 + y^2} + y.$

17. $2y' = \frac{y^2}{x^2} + 8\frac{y}{x} + 8.$

18. $xy' = \frac{3y^3 + 10yx^2}{2y^2 + 5x^2}.$

19. $y' = \frac{x^2 + 3xy - y^2}{3x^2 - 2xy}.$

20. $xy' = 3\sqrt{2x^2 + y^2} + y.$

21. $y' = \frac{y^2}{x^2} + 8\frac{y}{x} + 12.$

22. $xy' = \frac{3y^3 + 12yx^2}{2y^2 + 6x^2}.$

23. $y' = \frac{x^2 + xy - 3y^2}{x^2 - 4xy}.$

24. $xy' = 2\sqrt{3x^2 + y^2} + y.$

25. $4y' = \frac{y^2}{x^2} + 10\frac{y}{x} + 5.$

26. $xy' = \frac{3y^3 + 14yx^2}{2y^2 + 7x^2}.$

$$27. y' = \frac{x^2 + xy - 5y^2}{x^2 - 6xy}.$$

$$28. xy' = 4\sqrt{x^2 + y^2} + y.$$

$$29. 3y' = \frac{y^2}{x^2} + 10\frac{y}{x} + 10.$$

$$30. xy' = 4\sqrt{2x^2 + y^2} + y.$$

Задача 3. Найти общее решение дифференциального уравнения.

$$1. y' = \frac{x + 2y - 3}{2x - 2}.$$

$$2. y' = \frac{x + y - 2}{2x - 2}.$$

$$3. y' = \frac{3y - x - 4}{3x + 3}.$$

$$4. y' = \frac{2y - 2}{x + y - 2}.$$

$$5. y' = \frac{x + y - 2}{3x - y - 2}.$$

$$6. y' = \frac{2x + y - 3}{x - 1}.$$

$$7. y' = \frac{x + y - 8}{3x - y - 8}.$$

$$8. y' = \frac{x + 3y + 4}{3x - 6}.$$

$$9. y' = \frac{3y + 3}{2x + y - 1}.$$

$$10. y' = \frac{x + 2y - 3}{4x - y - 3}.$$

$$11. y' = \frac{x - 2y + 3}{-2x - 2}.$$

$$12. y' = \frac{x + 8y - 9}{10x - y - 9}.$$

$$13. y' = \frac{2x + 3y - 5}{5x - 5}.$$

$$14. y' = \frac{4y - 8}{3x + 2y - 7}.$$

$$15. y' = \frac{x + 3y - 4}{5x - y - 4}.$$

$$16. y' = \frac{y - 2x + 3}{x - 1}.$$

$$17. y' = \frac{x + 2y - 3}{x - 1}.$$

$$18. y' = \frac{3x + 2y - 1}{x + 1}.$$

$$19. y' = \frac{5y + 5}{4x + 3y - 1}.$$

$$20. y' = \frac{x + 4y - 5}{6x - y - 5}.$$

$$21. y' = \frac{x + y + 2}{x + 1}.$$

$$22. y' = \frac{2x + y - 3}{4x - 4}.$$

$$23. y' = \frac{2x + y - 3}{2x - 2}.$$

$$24. y' = \frac{y}{2x + 2y - 2}.$$

$$25. y' = \frac{x + 5y - 6}{7x - y - 6}.$$

$$26. y' = \frac{x + y - 4}{x - 2}.$$

$$27. y' = \frac{2x + y - 1}{2x - 2}.$$

$$28. y' = \frac{3y - 2x + 1}{3x + 3}.$$

$$29. y' = \frac{6y - 6}{5x + 4y - 9}.$$

$$30. y' = \frac{x + 6y - 7}{8x - y - 7}.$$

Задача 4. Найти решение задачи Коши.

$$1. y' - y/x = x^2, \quad y(1) = 0.$$

$$2. y' - y \operatorname{ctg} x = 2x \sin x, \quad y(\pi/2) = 0.$$

$$3. y' + y \cos x = \frac{1}{2} \sin 2x, \quad y(0) = 0.$$

$$4. y' + y \operatorname{tg} x = \cos^2 x, \quad y(\pi/4) = 1/2.$$

$$5. y' - \frac{y}{x+2} = x^2 + 2x, \quad y(-1) = 3/2.$$

$$6. y' - \frac{1}{x+1} y = e^x (x+1), \quad y(0) = 1.$$

$$7. y' - \frac{y}{x} = x \sin x, \quad y\left(\frac{\pi}{2}\right) = 1.$$

$$8. y' + \frac{y}{x} = \sin x, \quad y(\pi) = \frac{1}{\pi}.$$

9. $y' + \frac{y}{2x} = x^2$, $y(1) = 1$.
10. $y' + \frac{2x}{1+x^2}y = \frac{2x^2}{1+x^2}$, $y(0) = \frac{2}{3}$.
11. $y' - \frac{2x-5}{x^2}y = 5$, $y(2) = 4$.
12. $y' + \frac{y}{x} = \frac{x+1}{x}e^x$, $y(1) = e$.
13. $y' - \frac{y}{x} = -2\frac{\ln x}{x}$, $y(1) = 1$.
14. $y' - \frac{y}{x} = -\frac{12}{x^3}$, $y(1) = 4$.
15. $y' + \frac{2}{x}y = x^3$, $y(1) = -5/6$.
16. $y' + \frac{y}{x} = 3x$, $y(1) = 1$.
17. $y' - \frac{2xy}{1+x^2} = 1+x^2$, $y(1) = 3$.
18. $y' + \frac{1-2x}{x^2}y = 1$, $y(1) = 1$.
19. $y' + \frac{3y}{x} = \frac{2}{x^3}$, $y(1) = 1$.
20. $y' + 2xy = -2x^3$, $y(1) = e^{-1}$.
21. $y' + \frac{xy}{2(1-x^2)} = \frac{x}{2}$, $y(0) = \frac{2}{3}$.
22. $y' + xy = -x^3$, $y(0) = 3$.
23. $y' - \frac{2}{x+1}y = e^x(x+1)^2$, $y(0) = 1$.
24. $y' + 2xy = xe^{-x^2} \sin x$, $y(0) = 1$.
25. $y' - 2y/(x+1) = (x+1)^3$, $y(0) = 1/2$.
26. $y' - y \cos x = -\sin 2x$, $y(0) = 3$.
27. $y' - 4xy = -4x^3$, $y(0) = -1/2$.
28. $y' - \frac{y}{x} = -\frac{\ln x}{x}$, $y(1) = 1$.
29. $y' - 3x^2y = x^2(1+x^3)/3$, $y(0) = 0$.
30. $y' - y \cos x = \sin 2x$, $y(0) = -1$.

Задача 5. Решить задачу Коши.

- $y^2 dx + (x + e^{2/y}) dy = 0$, $y|_{x=e} = 2$.
- $(y^4 e^y + 2x)y' = y$, $y|_{x=0} = 1$.
- $y^2 dx + (xy - 1) dy = 0$, $y|_{x=1} = e$.
- $2(4y^2 + 4y - x)y' = 1$, $y|_{x=0} = 0$.
- $(\cos 2y \cos^2 y - x)y' = \sin y \cos y$, $y|_{x=1/4} = \pi/3$.
- $(x \cos^2 y - y^2)y' = y \cos^2 y$, $y|_{x=\pi} = \pi/4$.
- $e^{y^2} (dx - 2xy dy) = y dy$, $y|_{x=0} = 0$.
- $(104y^3 - x)y' = 4y$, $y|_{x=8} = 1$.
- $dx + (xy - y^3) dy = 0$, $y|_{x=-1} = 0$.

10. $(3y \cos 2y - 2y^2 \sin 2y - 2x)y' = y, \quad y|_{x=16} = \pi/4.$
11. $8(4y^3 + xy - y)y' = 1, \quad y|_{x=0} = 0.$
12. $(2 \ln y - \ln^2 y)dy = ydx - xdy, \quad y|_{x=4} = e^2.$
13. $2(x + y^4)y' = y, \quad y|_{x=-2} = -1.$
14. $y^3(y - 1)dx + 3xy^2(y - 1)dy = (y + 2)dy, \quad y|_{x=1/4} = 2.$
15. $2y^2dx + (x + e^{1/y})dy = 0, \quad y|_{x=e} = 1.$
16. $(xy + \sqrt{y})dy + y^2dx = 0, \quad y|_{x=-1/2} = 4.$
17. $\sin 2ydx = (\sin^2 2y - 2\sin^2 y + 2x)dy, \quad y|_{x=-1/2} = \pi/4.$
18. $(y^2 + 2y - x)y' = 1, \quad y|_{x=2} = 0.$
19. $2y\sqrt{y}dx - (6x\sqrt{y} + 7)dy = 0, \quad y|_{x=-4} = 1.$
20. $dx = (\sin y + 3\cos y + 3x)dy, \quad y|_{x=e^{\pi/2}} = \pi/2.$
21. $2(\cos^2 y \cdot \cos 2y - x)y' = \sin 2y, \quad y|_{x=3/2} = 5\pi/4.$
22. $\operatorname{ch} ydx = (1 + x \operatorname{sh} x)dy, \quad y|_{x=1} = \ln 2.$
23. $(13y^3 - x)y' = 4y, \quad y|_{x=5} = 1.$
24. $y^2(y^2 + 4)dx + 2xy(y^2 + 4)dy = 2dy, \quad y|_{x=\pi/8} = 2.$
25. $(x + \ln^2 y - \ln y)y' = y/2, \quad y|_{x=2} = 1.$
26. $(2xy + \sqrt{y})dy + 2y^2dx = 0, \quad y|_{x=-1/2} = 1.$
27. $ydx + (2x - 2\sin^2 y - y \sin 2y)dy = 0, \quad y|_{x=3/2} = \pi/4.$
28. $2(y^3 - y + xy)dy = dx, \quad y|_{x=-2} = 0.$
29. $(2y + x \operatorname{tg} y - y^2 \operatorname{tg} y)dy = dx, \quad y|_{x=0} = \pi.$
30. $4y^2dx + (e^{1/(2y)} + x)dy = 0, \quad y|_{x=e} = 1/2.$

Задача 6. Найти решение задачи Коши.

1. $y' + xy = (1 + x)e^{-x} y^2, \quad y(0) = 1.$

2. $xy' + y = 2y^2 \ln x$, $y(1) = 1/2$.
3. $2(xy' + y) = xy^2$, $y(1) = 2$.
4. $y' + 4x^3y = 4(x^3 + 1)e^{-4x}y^2$, $y(0) = 1$.
5. $xy' - y = -y^2(\ln x + 2)\ln x$, $y(1) = 1$.
6. $2(y' + xy) = (1 + x)e^{-x}y^2$, $y(0) = 2$.
7. $3(xy' + y) = y^2 \ln x$, $y(1) = 3$.
8. $2y' + y \cos x = y^{-1} \cos x(1 + \sin x)$, $y(0) = 1$.
9. $y' + 4x^3y = 4y^2 e^{4x}(1 - x^3)$, $y(0) = -1$.
10. $3y' + 2xy = 2xy^{-2} e^{-2x^2}$, $y(0) = -1$.
11. $2xy' - 3y = -(5x^2 + 3)y^3$, $y(1) = 1/\sqrt{2}$.
12. $3xy' + 5y = (4x - 5)y^4$, $y(1) = 1$.
13. $2y' + 3y \cos x = e^{2x}(2 + 3\cos x)y^{-1}$, $y(0) = 1$.
14. $3(xy' + y) = xy^2$, $y(1) = 3$.
15. $y' - y = 2xy^2$, $y(0) = 1/2$.
16. $2xy' - 3y = -(20x^2 + 12)y^3$, $y(1) = 1/2\sqrt{2}$.
17. $y' + 2xy = 2x^3y^3$, $y(0) = \sqrt{2}$.
18. $xy' + y = y^2 \ln x$, $y(1) = 1$.
19. $2y' + 3y \cos x = (8 + 12 \cos x)e^{2x}y^{-1}$, $y(0) = 2$.
20. $4y' + x^3y = (x^3 + 8)e^{-2x}y^2$, $y(0) = 1$.
21. $8xy' - 12y = -(5x^2 + 3)y^3$, $y(1) = \sqrt{2}$.
22. $2(y' + y) = xy^2$, $y(0) = 2$.
23. $y' + xy = (x - 1)e^x y^2$, $y(0) = 1$.
24. $2y' + 3y \cos x = -e^{-2x}(2 + 3\cos x)y^{-1}$, $y(0) = 1$.
25. $y' - y = xy^2$, $y(0) = 1$.
26. $2(xy' + y) = y^2 \ln x$, $y(1) = 2$.

27. $y' + y = xy^2$, $y(0) = 1$.
28. $y' + 2y \operatorname{cth} x = y^2 \operatorname{ch} x$, $y(1) = 1/\operatorname{sh} 1$.
29. $2(y' + xy) = (x-1)e^x y^2$, $y(0) = 2$.
30. $y' - y \operatorname{tg} x = -(2/3)y^4 \sin x$, $y(0) = 1$.
31. $xy' + y = xy^2$, $y(1) = 1$.

Задача 7. Найти общее решение дифференциального уравнения.

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|---|--|
| 1. $y'''x \ln x = y''$. | 2. $xy''' + y'' = 1$. |
| 3. $2xy''' = y''$. | 4. $xy''' + y'' = x + 1$. |
| 5. $\operatorname{tg} x \cdot y'' - y' + \frac{1}{\sin x} = 0$. | 6. $x^2 y'' + xy' = 1$. |
| 7. $y''' \operatorname{ctg} 2x + 2y'' = 0$. | 8. $x^3 y''' + x^2 y'' = 1$. |
| 9. $\operatorname{tg} x \cdot y''' = 2y''$. | 10. $y''' \operatorname{cth} 2x = 2y''$. |
| 11. $x^4 y'' + x^3 y' = 1$. | 12. $xy''' + 2y'' = 0$. |
| 13. $(1+x^2)y'' + 2xy' = x^3$. | 14. $x^5 y''' + x^4 y'' = 1$. |
| 15. $xy''' - y'' + \frac{1}{x} = 0$. | 16. $xy''' + y'' + x = 0$. |
| 17. $\operatorname{th} x \cdot y^{IV} = y'''$. | 18. $xy''' + y'' = \sqrt{x}$. |
| 19. $y''' \operatorname{tg} x = y'' + 1$. | 20. $y''' \operatorname{tg} 5x = 5y''$. |
| 21. $y''' \operatorname{th} 7x = 7y''$. | 22. $x^3 y''' + x^2 y'' = \sqrt{x}$. |
| 23. $\operatorname{cth} x \cdot y'' - y' + \frac{1}{\operatorname{ch} x} = 0$. | 24. $(x+1)y''' + y'' = (x+1)$. |
| 25. $(1 + \sin x)y''' = \cos x \cdot y''$. | 26. $xy''' + y'' = \frac{1}{\sqrt{x}}$. |
| 27. $-xy''' + 2y'' = \frac{2}{x^2}$. | 28. $\operatorname{cth} xy'' + y' = \operatorname{ch} x$. |
| 29. $x^4 y'' + x^3 y' = 4$. | 30. $y'' + \frac{2x}{x^2 + 1} y' = 2x$. |

Задача 8. Найти решение задачи Коши.

1. $4y^3y'' = y^4 - 1$, $y(0) = \sqrt{2}$, $y'(0) = 1/(2\sqrt{2})$.

2. $y'' = 128y^3$, $y(0) = 1$, $y'(0) = 8$.

3. $y''y^3 + 64 = 0$, $y(0) = 4$, $y'(0) = 2$.

4. $y'' + 2\sin y \cos^3 y = 0$, $y(0) = 0$, $y'(0) = 1$.

5. $y'' = 32\sin^3 y \cos y$, $y(1) = \pi/2$, $y'(1) = 4$.

6. $y'' = 98y^3$, $y(1) = 1$, $y'(1) = 7$.

7. $y''y^3 + 49 = 0$, $y(3) = -7$, $y'(3) = -1$.

8. $4y^3y'' = 16y^4 - 1$, $y(0) = \sqrt{2}/2$, $y'(0) = 1/\sqrt{2}$.

9. $y'' + 8\sin y \cos^3 y = 0$, $y(0) = 0$, $y'(0) = 2$.

10. $y'' = 72y^3$, $y(2) = 1$, $y'(2) = 6$.

11. $y''y^3 + 36 = 0$, $y(0) = 3$, $y'(0) = 2$.

12. $y'' = 18\sin^3 y \cos y$, $y(1) = \pi/2$, $y'(1) = 3$.

13. $4y^3y'' = y^4 - 16$, $y(0) = 2\sqrt{2}$, $y'(0) = 1/\sqrt{2}$.

14. $y'' = 50y^3$, $y(3) = 1$, $y'(3) = 5$.

15. $y''y^3 + 25 = 0$, $y(2) = -5$, $y'(2) = -1$.

16. $y'' + 18\sin y \cos^3 y = 0$, $y(0) = 0$, $y'(0) = 3$.

17. $y'' = 8\sin^3 y \cos y$, $y(1) = \pi/2$, $y'(1) = 2$.

18. $y'' = 32y^3$, $y(4) = 1$, $y'(4) = 4$.

19. $y''y^3 + 16 = 0$, $y(1) = 2$, $y'(1) = 2$.

20. $y'' + 32\sin y \cos^3 y = 0$, $y(0) = 0$, $y'(0) = 4$.

21. $y'' = 50\sin^3 y \cos y$, $y(1) = \pi/2$, $y'(1) = 5$.

22. $y'' = 18y^3$, $y(1) = 1$, $y'(1) = 3$.

23. $y''y^3 + 9 = 0$, $y(1) = 1$, $y'(1) = 3$.

24. $y^3y'' = 4(y^4 - 1)$, $y(0) = \sqrt{2}$, $y'(0) = \sqrt{2}$.

25. $y'' + 50\sin y \cos^3 y = 0$, $y(0) = 0$, $y'(0) = 5$.

26. $y'' = 3y^3$, $y(0) = 1$, $y'(0) = 2$.
27. $y''y^3 + 4 = 0$, $y(0) = -1$, $y'(0) = -2$.
28. $y'' = 2\sin^3 y \cos y$, $y(1) = \pi/2$, $y'(1) = 1$.
29. $y^3y'' = y^4 - 16$, $y(0) = 2\sqrt{2}$, $y'(0) = \sqrt{2}$.
30. $y'' = 2y^3$, $y(-1) = 1$, $y'(-1) = 1$.

Задача 9. Найти общее решение дифференциального уравнения.

- | | |
|--|---|
| 1. $y''' + 3y'' + 2y' = 1 - x^2$. | 2. $y''' - y'' = 6x^2 + 3x$. |
| 3. $y''' - y' = x^2 + x$. | 4. $y^{IV} - 3y''' + 3y'' - y' = 2x$. |
| 5. $y^{IV} - y''' = 5(x + 2)^2$. | 6. $y^{IV} - 2y''' + y'' = 2x(1 - x)$. |
| 7. $y^{IV} + 2y''' + y'' = x^2 + x - 1$. | 8. $y^V - y^{IV} = 2x + 3$. |
| 9. $3y^{IV} + y''' = 6x - 1$. | 10. $y^{IV} + 2y''' + y'' = 4x^2$. |
| 11. $y''' + y'' = 5x^2 - 1$. | 12. $y^{IV} + 4y''' + 4y'' = x - x^2$. |
| 13. $7y''' - y'' = 12x$. | 14. $y''' + 3y'' + 2y' = 3x^2 + 2x$. |
| 15. $y''' - y' = 3x^2 - 2x + 1$. | 16. $y''' - y'' = 4x^2 - 3x + 2$. |
| 17. $y^{IV} - 3y''' + 3y'' - y' = x - 3$. | 18. $y^{IV} + 2y''' + y'' = 12x^2 - 6x$. |
| 19. $y''' - 4y'' = 32 - 384x^2$. | 20. $y^{IV} + 2y''' + y'' = 2 - 3x^2$. |
| 21. $y''' + y'' = 49 - 24x^2$. | 22. $y''' - 2y'' = 3x^2 + x - 4$. |
| 23. $y''' - 13y'' + 12y' = x - 1$. | 24. $y^{IV} + y''' = x$. |
| 25. $y''' - y'' = 6x + 5$. | 26. $y''' + 3y'' + 2y' = x^2 + 2x + 3$. |
| 27. $y''' - 5y'' + 6y' = (x - 1)^2$. | 28. $y^{IV} - 6y''' + 9y'' = 3x - 1$. |
| 29. $y''' - 13y'' + 12y' = 18x^2 - 39$. | 30. $y^{IV} + y''' = 12x + 6$. |

Задача 10. Найти общее решение дифференциального уравнения.

- | | |
|--|---|
| 1. $y''' - 4y'' + 5y' - 2y = (16 - 12x)e^{-x}$. | 2. $y''' - 3y'' + 2y' = (1 - 2x)e^x$. |
| 3. $y''' - y'' - y' + y = (3x + 7)e^{2x}$. | 4. $y''' - 2y'' + y' = (2x + 5)e^{2x}$. |
| 5. $y''' - 3y'' + 4y = (18x - 21)e^{-x}$. | 6. $y''' - 5y'' + 8y' - 4y = (2x - 5)e^x$. |

7. $y''' - 4y'' + 4y' = (x - 1)e^x$.
8. $y''' + 2y'' + y' = (18x + 21)e^{2x}$.
9. $y''' + y'' - y' - y = (8x + 4)e^x$.
10. $y''' - 3y'' - 2y = -4x \cdot e^x$.
11. $y''' - 3y'' + 2y = (4x + 9)e^{2x}$.
12. $y''' + 4y'' + 5y' + 2y = (12x + 16)e^x$.
13. $y''' - y'' - 2y' = (6x - 11)e^{-x}$.
14. $y''' + y'' - 2y' = (6x + 5)e^x$.
15. $y''' + 4y'' + 4y' = (9x + 15)e^x$.
16. $y''' - 3y'' - y' + 3y = (4 - 8x)e^x$.
17. $y''' - y'' - 4y' + 4y = (7 - 6x)e^x$.
18. $y''' + 3y'' + 2y' = (1 - 2x)e^{-x}$.
19. $y''' - 5y'' + 7y' - 3y = (20 - 16x)e^{-x}$.
20. $y''' - 4y'' + 3y' = -4x \cdot e^{-x}$.
21. $y''' - 5y'' + 3y' + 9y = (32x - 32)e^{-x}$.
22. $y''' - 6y'' + 9y' = 4x \cdot e^x$.
23. $y''' - 7y'' + 15y' - 9y = (8x - 12)e^x$.
24. $y''' - y'' - 5y' - 3y = -(8x + 4)e^x$.
25. $y''' + 5y'' + 7y' + 3y = (16x + 20)e^x$.
26. $y''' - 2y'' - 3y' = (8x - 14)e^{-x}$.
27. $y''' + 2y'' - 3y' = (8x + 6)e^x$.
28. $y''' + 6y'' + 9y' = (16x + 24)e^x$.
29. $y''' - y'' - 9y' + 9y = (12 - 16x)e^x$.
30. $y''' + 4y'' + 3y' = 4(1 - x)e^{-x}$.

Задача 11. Найти сумму ряда.

1. $\sum_{n=1}^{\infty} \frac{6}{9n^2 + 12n - 5}$.
2. $\sum_{n=2}^{\infty} \frac{24}{9n^2 - 12n - 5}$.
3. $\sum_{n=1}^{\infty} \frac{6}{9n^2 + 6n - 8}$.
4. $\sum_{n=1}^{\infty} \frac{9}{9n^2 + 21n - 8}$.
5. $\sum_{n=0}^{\infty} \frac{2}{4n^2 + 8n + 3}$.
6. $\sum_{n=1}^{\infty} \frac{14}{49n^2 - 28n - 45}$.
7. $\sum_{n=1}^{\infty} \frac{3}{9n^2 + 3n - 2}$.
8. $\sum_{n=1}^{\infty} \frac{7}{49n^2 - 7n - 12}$.
9. $\sum_{n=2}^{\infty} \frac{1}{n^2 + n - 2}$.
10. $\sum_{n=1}^{\infty} \frac{14}{49n^2 - 14n - 48}$.
11. $\sum_{n=1}^{\infty} \frac{6}{36n^2 - 24n - 5}$.
12. $\sum_{n=1}^{\infty} \frac{14}{49n^2 - 84n - 13}$.
13. $\sum_{n=1}^{\infty} \frac{4}{4n^2 + 4n - 3}$.
14. $\sum_{n=1}^{\infty} \frac{7}{49n^2 + 35n - 6}$.
15. $\sum_{n=1}^{\infty} \frac{9}{9n^2 + 3n - 20}$.
16. $\sum_{n=1}^{\infty} \frac{14}{49n^2 - 42n - 40}$.
17. $\sum_{n=1}^{\infty} \frac{8}{16n^2 - 8n - 15}$.
18. $\sum_{n=1}^{\infty} \frac{7}{49n^2 - 21n - 10}$.
19. $\sum_{n=1}^{\infty} \frac{5}{25n^2 + 5n - 6}$.
20. $\sum_{n=1}^{\infty} \frac{6}{4n^2 - 9}$.
21. $\sum_{n=1}^{\infty} \frac{7}{49n^2 - 35n - 6}$.
22. $\sum_{n=2}^{\infty} \frac{1}{n^2 + n - 2}$.
23. $\sum_{n=2}^{\infty} \frac{12}{36n^2 + 12n - 35}$.
24. $\sum_{n=1}^{\infty} \frac{24}{49n^2 + 21n - 10}$.

25. $\sum_{n=1}^{\infty} \frac{3}{9n^2 - 3n - 2}.$

26. $\sum_{n=1}^{\infty} \frac{5}{25n^2 - 5n - 6}.$

27. $\sum_{n=1}^{\infty} \frac{8}{16n^2 + 8n - 15}.$

28. $\sum_{n=1}^{\infty} \frac{14}{49n^2 - 56n - 33}.$

29. $\sum_{n=1}^{\infty} \frac{12}{36n^2 - 12n - 35}.$

30. $\sum_{n=1}^{\infty} \frac{7}{49n^2 + 7n - 12}.$

Задача 12. Найти сумму ряда.

1. $\sum_{n=3}^{\infty} \frac{4-5n}{n(n-1)(n-2)}.$

2. $\sum_{n=1}^{\infty} \frac{n+6}{n(n+3)(n+2)}.$

3. $\sum_{n=1}^{\infty} \frac{5n+3}{n(n+1)(n+3)}.$

4. $\sum_{n=3}^{\infty} \frac{4n-2}{(n^2-1)(n-2)}.$

5. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+3)}.$

6. $\sum_{n=3}^{\infty} \frac{3n-5}{n(n^2-1)}.$

7. $\sum_{n=1}^{\infty} \frac{1}{n(n+2)(n+3)}.$

8. $\sum_{n=3}^{\infty} \frac{1}{n(n^2-4)}.$

9. $\sum_{n=1}^{\infty} \frac{3n-2}{n(n+1)(n+2)}.$

10. $\sum_{n=3}^{\infty} \frac{n+2}{n(n-1)(n-2)}.$

11. $\sum_{n=3}^{\infty} \frac{5n-2}{(n-1)n(n+2)}.$

12. $\sum_{n=1}^{\infty} \frac{2}{(n+2)(n+1)n}.$

13. $\sum_{n=1}^{\infty} \frac{3n+2}{n(n+1)(n+2)}.$

14. $\sum_{n=3}^{\infty} \frac{n+5}{(n^2-1)(n+2)}.$

15. $\sum_{n=3}^{\infty} \frac{8n-10}{(n-1)(n-2)(n+1)}.$

16. $\sum_{n=3}^{\infty} \frac{3n-1}{n(n^2-1)}.$

17. $\sum_{n=3}^{\infty} \frac{n-4}{n(n-1)(n-2)}.$

18. $\sum_{n=1}^{\infty} \frac{5n+9}{n(n+1)(n+3)}.$

19. $\sum_{n=2}^{\infty} \frac{5n-2}{(n-1)n(n+2)}.$

20. $\sum_{n=1}^{\infty} \frac{n-1}{n(n+1)(n+2)}.$

21. $\sum_{n=1}^{\infty} \frac{3n+4}{n(n+1)(n+2)}.$

22. $\sum_{n=3}^{\infty} \frac{2-n}{n(n+1)(n+2)}.$

23. $\sum_{n=1}^{\infty} \frac{n+6}{n(n+1)(n+2)}.$

24. $\sum_{n=3}^{\infty} \frac{n-2}{(n-1)n(n+1)}.$

25. $\sum_{n=2}^{\infty} \frac{1}{n(n^2-1)}.$

26. $\sum_{n=1}^{\infty} \frac{1-n}{n(n+1)(n+3)}.$

27. $\sum_{n=3}^{\infty} \frac{3n+1}{(n-1)n(n+1)}.$

28. $\sum_{n=1}^{\infty} \frac{4-n}{n(n+1)(n+2)}.$

29. $\sum_{n=3}^{\infty} \frac{4}{n(n-1)(n-2)}.$

30. $\sum_{n=1}^{\infty} \frac{3-n}{(n+3)(n+1)n}.$

Задача 13. Исследовать на сходимость ряд.

1. $\sum_{n=2}^{\infty} \frac{n+1}{2^n(n-1)!}.$

2. $\sum_{n=1}^{\infty} \frac{(n!)^2}{2^{n^2}}.$

3. $\sum_{n=1}^{\infty} \frac{2^{n+1}(n^3+1)}{(n+1)!}.$

4. $\sum_{n=1}^{\infty} \frac{10^n 2n!}{(2n)!}.$

5. $\sum_{n=1}^{\infty} \frac{(2n+2)!}{3n+5} \cdot \frac{1}{2^n}.$

6. $\sum_{n=1}^{\infty} \frac{n+5}{n!} \sin \frac{2}{3^n}.$

$$7. \sum_{n=1}^{\infty} \frac{\arctg \frac{5}{n}}{n!}.$$

$$8. \sum_{n=1}^{\infty} \frac{n^n}{3^n n!}.$$

$$9. \sum_{n=1}^{\infty} \frac{n}{(2n)!} \operatorname{tg} \frac{1}{5^n}.$$

$$10. \sum_{n=1}^{\infty} \frac{6^n (n^2 - 1)}{n!}.$$

$$11. \sum_{n=1}^{\infty} \frac{n^2}{(n+2)!}.$$

$$12. \sum_{n=1}^{\infty} \frac{n^n}{(n!)^2}.$$

$$13. \sum_{n=1}^{\infty} \frac{7^{2n}}{(2n-1)!}.$$

$$14. \sum_{n=1}^{\infty} \frac{n!}{(3n)!}.$$

$$15. \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{3^n (n+1)!}.$$

$$16. \sum_{n=1}^{\infty} \frac{n!}{n^{n-1}}. \quad ?$$

$$17. \sum_{n=1}^{\infty} \frac{(n!)^2}{(3^n + 1)(2n)!}.$$

$$18. \sum_{n=1}^{\infty} n \sin \frac{\pi}{2^n}.$$

$$19. \sum_{n=1}^{\infty} \frac{(n+1)!}{n^n}.$$

$$20. \sum_{n=1}^{\infty} \frac{5^n \sqrt[3]{n^2}}{(n+1)!}.$$

$$21. \sum_{n=1}^{\infty} \frac{2^n n!}{n^n}.$$

$$22. \sum_{n=1}^{\infty} \frac{5^n (n+1)!}{(2n)!}.$$

$$23. \sum_{n=1}^{\infty} \frac{3^n}{(n+2)! 4^n}.$$

$$24. \sum_{n=1}^{\infty} \frac{3 \cdot 5 \cdot 7 \dots (2n+1)}{2 \cdot 5 \cdot 8 \dots (3n-1)}.$$

$$25. \sum_{n=1}^{\infty} \frac{1 \cdot 4 \cdot 7 \dots (3n-2)}{7 \cdot 9 \cdot 11 \dots (2n+5)}.$$

$$26. \sum_{n=1}^{\infty} \frac{2n!}{\sqrt{2^n + 3}}.$$

$$27. \sum_{n=1}^{\infty} \frac{(3n+2)!}{10^n n^2}.$$

$$28. \sum_{n=2}^{\infty} \frac{4^{n-1} \sqrt{n^2 + 5}}{(n-1)!}.$$

$$29. \sum_{n=1}^{\infty} \frac{n! \sqrt[3]{n}}{3^n + 2}.$$

$$30. \sum_{n=1}^{\infty} \frac{n!(2n+1)!}{(3n)!}.$$

Задача 14. Исследовать на сходимость ряд.

$$1. \sum_{n=2}^{\infty} \frac{1}{3^n} \left(\frac{n}{n+1} \right)^{-n^2}.$$

$$2. \sum_{n=1}^{\infty} \frac{1}{4^n} \left(1 + \frac{1}{n} \right)^{n^2}.$$

$$3. \sum_{n=1}^{\infty} \left(\frac{2n^2 + 1}{n^2 + 1} \right)^{n^2}.$$

$$4. \sum_{n=1}^{\infty} n^4 \left(\frac{2n}{3n+5} \right)^n.$$

$$5. \sum_{n=1}^{\infty} \left(\frac{2n+1}{3n-2} \right)^{n^2}.$$

$$6. \sum_{n=1}^{\infty} \left(\frac{2n+2}{3n+1} \right)^n (n+1)^3.$$

$$7. \sum_{n=1}^{\infty} \left(\frac{4n-3}{5n+1} \right)^{n^3}.$$

$$8. \sum_{n=1}^{\infty} \left(\frac{n}{10n+5} \right)^{n^2}.$$

$$9. \sum_{n=1}^{\infty} n \arcsin^n \frac{\pi}{4n}.$$

$$10. \sum_{n=1}^{\infty} \left(\frac{n+2}{3n-1} \right)^{n^2}.$$

$$11. \sum_{n=1}^{\infty} \left(\frac{n-1}{n} \right)^n \frac{n}{5^n}.$$

$$12. \sum_{n=1}^{\infty} \left(\frac{2n+3}{n+1} \right)^{n^2}.$$

$$13. \sum_{n=1}^{\infty} \left(\frac{3n+2}{4n-1} \right)^n (n-1)^2.$$

$$14. \sum_{n=2}^{\infty} \left(\frac{n+1}{2n-3} \right)^{n^2}.$$

$$15. \sum_{n=1}^{\infty} \left(\frac{n}{3n+1} \right)^{2n+1}.$$

$$16. \sum_{n=1}^{\infty} \left(\frac{2n-1}{3n+1} \right)^{n/2}.$$

$$17. \sum_{n=1}^{\infty} \frac{2^{n+1}}{n^n}.$$

$$18. \sum_{n=1}^{\infty} n^2 \sin^n \frac{\pi}{2n}.$$

$$\begin{array}{lll}
19. \sum_{n=2}^{\infty} \frac{n^3}{(\ln n)^n} & 20. \sum_{n=1}^{\infty} \left(\frac{n}{3n-1} \right)^{n^3} & 21. \sum_{n=1}^{\infty} n^3 \operatorname{arctg}^n \frac{\pi}{3n} \\
22. \sum_{n=1}^{\infty} \frac{n^5 3^n}{(2n+1)^n} & 23. \sum_{n=1}^{\infty} 2^{n-1} e^{-n} & 24. \sum_{n=1}^{\infty} n \left(\frac{3n-1}{4n+2} \right)^{2n} \\
25. \sum_{n=1}^{\infty} \left(\frac{2n}{4n+3} \right)^{n^2} & 26. \sum_{n=1}^{\infty} \frac{n^{n+2}}{(2n^2+1)^{n/2}} & 27. \sum_{n=1}^{\infty} \sqrt{n} \left(\frac{n}{3n-1} \right)^{2n} \\
28. \sum_{n=1}^{\infty} \left(\frac{n+1}{n} \right)^{n^2} \frac{1}{2^n} & 29. \sum_{n=1}^{\infty} \frac{n \cdot 3^{n+2}}{5^n} & 30. \sum_{n=2}^{\infty} \sqrt[3]{n} \left(\frac{n-2}{2n+1} \right)^{3n}
\end{array}$$

Задача 15. Исследовать на сходимость ряд.

$$\begin{array}{lll}
1. \sum_{n=2}^{\infty} \frac{1}{n \ln^2(3n+1)} & 2. \sum_{n=1}^{\infty} \frac{1}{n \ln^2(2n+1)} & 3. \sum_{n=1}^{\infty} \frac{1}{(2n+3) \ln^2(2n+1)} \\
4. \sum_{n=3}^{\infty} \frac{1}{(3n-5) \ln^2(4n-7)} & 5. \sum_{n=1}^{\infty} \frac{1}{(3n+4) \ln^2(5n+2)} & 6. \sum_{n=1}^{\infty} \frac{1}{(2n+1) \ln^2(n\sqrt{5}+2)} \\
7. \sum_{n=1}^{\infty} \frac{1}{(n\sqrt{2}+1) \ln^2(n\sqrt{3}+1)} & 8. \sum_{n=5}^{\infty} \frac{1}{(n-2) \ln(n-3)} & 9. \sum_{n=1}^{\infty} \frac{1}{(2n-1) \ln(2n)} \\
10. \sum_{n=1}^{\infty} \frac{1}{(n+1) \ln(2n)} & 11. \sum_{n=2}^{\infty} \frac{1}{(3n-1) \ln n} & 12. \sum_{n=2}^{\infty} \frac{1}{(2n-1) \ln(n+1)} \\
13. \sum_{n=2}^{\infty} \frac{1}{(2n-3) \ln(3n+1)} & 14. \sum_{n=2}^{\infty} \frac{1}{(n+2) \ln^2 n} & 15. \sum_{n=2}^{\infty} \frac{1}{(n+3) \ln^2(2n)} \\
16. \sum_{n=2}^{\infty} \frac{1}{(2n+3) \ln^2(n+1)} & 17. \sum_{n=3}^{\infty} \frac{1}{n \ln(n-1)} & 18. \sum_{n=2}^{\infty} \frac{1}{2n\sqrt{\ln(3n-1)}} \\
19. \sum_{n=5}^{\infty} \frac{1}{(n-2)\sqrt{\ln(n-3)}} & 20. \sum_{n=4}^{\infty} \frac{1}{(3n-1)\sqrt{\ln(n-2)}} & 21. \sum_{n=2}^{\infty} \frac{1}{(n+5) \ln^2(n+1)} \\
22. \sum_{n=2}^{\infty} \frac{1}{(n/3) \ln^2(n+7)} & 23. \sum_{n=2}^{\infty} \frac{n^2}{(n^3+1) \ln n} & 24. \sum_{n=3}^{\infty} \frac{n}{(n^2-3) \ln^2 n} \\
25. \sum_{n=4}^{\infty} \frac{1}{(n/3-1) \ln^2(n/2)} & 26. \sum_{n=2}^{\infty} \frac{n}{(n^2+5) \ln n} & 27. \sum_{n=2}^{\infty} \frac{3n}{(2n^2+3) \ln n} \\
28. \sum_{n=4}^{\infty} \frac{n+1}{(5n^2-9) \ln(n-2)} & 29. \sum_{n=3}^{\infty} \frac{2n+1}{(3n^2/2+2) \ln(n/2)} & 30. \sum_{n=2}^{\infty} \frac{n}{(n^2-1) \ln n}
\end{array}$$

Задача 16. Исследовать на сходимость ряд.

1. $\sum_{n=2}^{\infty} (-1)^n \frac{2n+1}{n(n+1)}$.
2. $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{n}{2n+1} \right)^n$.
3. $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\ln(n+1)}$.
4. $\sum_{n=3}^{\infty} \frac{(-1)^n}{n(\ln \ln n) \ln n}$.
5. $\sum_{n=1}^{\infty} \frac{(-1)^n 2n^2}{n^4 - n^2 + 1}$.
6. $\sum_{n=3}^{\infty} \frac{(-1)^n}{(n+1) \ln n}$.
7. $\sum_{n=3}^{\infty} \frac{(-1)^n}{n \ln(n+1)}$.
8. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n \sqrt[4]{2n+3}}$.
9. $\sum_{n=1}^{\infty} \frac{(-1)^n \sin \frac{\pi}{2\sqrt{n}}}{\sqrt{3n+1}}$.
10. $\sum_{n=1}^{\infty} (-1)^n \cos \frac{\pi}{6n}$.
11. $\sum_{n=1}^{\infty} \frac{\sin n}{n!}$.
12. $\sum_{n=3}^{\infty} \frac{(-1)^n}{n \ln(2n)}$.
13. $\sum_{n=1}^{\infty} (-1)^n \operatorname{tg} \frac{1}{n}$.
14. $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$.
15. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(n+1) 2^{2n}}$.
16. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\cos \frac{\pi}{3\sqrt{n}} \sqrt[3]{3n + \ln n}}$.
17. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(n+1)(3/2)^n}$.
18. $\sum_{n=1}^{\infty} (-1)^n \frac{2n-1}{3n}$.
19. $\sum_{n=1}^{\infty} \frac{(-1)^n (n+3)}{\ln(n+4)}$.
20. $\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{\sqrt{n^3}}$.
21. $\sum_{n=1}^{\infty} \frac{(-1)^n \operatorname{tg} \frac{\pi}{4\sqrt{n}}}{\sqrt{5n-1}}$.
22. $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1) 2^{2n+2}}$.
23. $\sum_{n=1}^{\infty} (-1)^n \frac{\sin(n\sqrt{n})}{n\sqrt{n}}$.
24. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n + \cos(2/\sqrt{n+4})}$.
25. $\sum_{n=1}^{\infty} (-1)^n \sin \frac{\pi}{2^n}$.
26. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + \sin^2 n}$.
27. $\sum_{n=1}^{\infty} (-1)^n \frac{\sin 3^n}{3^n}$.
28. $\sum_{n=1}^{\infty} (-1)^n \ln \left(1 + \frac{1}{n^2} \right)$.
29. $\sum_{n=1}^{\infty} (-1)^n \sin \frac{1}{n} \cdot \operatorname{tg} \frac{1}{n}$.
30. $\sum_{n=1}^{\infty} (-1)^n \left(1 - \cos \frac{1}{\sqrt{n}} \right)$.

Задача 17. Вычислить сумму ряда с точностью α .

1. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{3n^2}$, $\alpha = 0,01$.
2. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}$, $\alpha = 0,01$.
3. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{(2n)^3}$, $\alpha = 0,001$.
4. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{n!(2n+1)}$, $\alpha = 0,001$.
5. $\sum_{n=1}^{\infty} (-1)^n \frac{2n+1}{n^3(n+1)}$, $\alpha = 0,01$.
6. $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!}$, $\alpha = 0,0001$.

7.
$$\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n}{2^n}, \quad \alpha = 0,1.$$

8.
$$\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n^2}{3^n}, \quad \alpha = 0,1.$$

9.
$$\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n}{(2n-1)^2 (2n+1)^2}, \quad \alpha = 0,001.$$

10.
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!!}, \quad \alpha = 0,0001.$$

11.
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!!}, \quad \alpha = 0,001.$$

12.
$$\sum_{n=0}^{\infty} \left(-\frac{2}{5}\right)^n, \quad \alpha = 0,01.$$

13.
$$\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n}{7^n}, \quad \alpha = 0,0001.$$

14.
$$\sum_{n=0}^{\infty} \left(-\frac{2}{3}\right)^n, \quad \alpha = 0,1.$$

15.
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!}, \quad \alpha = 0,001.$$

16.
$$\sum_{n=0}^{\infty} \frac{(-1)^n}{3n!}, \quad \alpha = 0,01.$$

17.
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!2n}, \quad \alpha = 0,00001.$$

18.
$$\sum_{n=1}^{\infty} \frac{(-1)^n \cdot (2n+1)}{(2n)!n!}, \quad \alpha = 0,001.$$

19.
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n \cdot n!}, \quad \alpha = 0,001.$$

20.
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n \cdot n!}, \quad \alpha = 0,001.$$

21.
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!n!}, \quad \alpha = 0,00001.$$

22.
$$\sum_{n=0}^{\infty} \frac{\cos \pi n}{3^n (n+1)}, \quad \alpha = 0,001.$$

23.
$$\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n (2n+1)}, \quad \alpha = 0,001.$$

24.
$$\sum_{n=1}^{\infty} \frac{\sin(\pi/2 + \pi n)}{n^3}, \quad \alpha = 0,01.$$

25.
$$\sum_{n=0}^{\infty} \frac{(-1)^n \cdot 2^n}{(n+1)^n}, \quad \alpha = 0,001.$$

26.
$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)^n}, \quad \alpha = 0,001.$$

27.
$$\sum_{n=1}^{\infty} \frac{\sin(\pi/2 + \pi n)}{n^3 + 1}, \quad \alpha = 0,01.$$

28.
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3 (n+3)}, \quad \alpha = 0,01.$$

29.
$$\sum_{n=0}^{\infty} \frac{\cos(\pi n)}{(n^3 + 1)^2}, \quad \alpha = 0,001.$$

30.
$$\sum_{n=0}^{\infty} \frac{(-1)^n}{1+n^2}, \quad \alpha = 0,01.$$

Задача 18. Найти область сходимости ряда.

1.
$$\sum_{n=1}^{\infty} \frac{(n-2)^3 (x+3)^{2n}}{2n+3}.$$

2.
$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{(n+1)5^n}.$$

3.
$$\sum_{n=1}^{\infty} \frac{(x-1)^{2n}}{n9^n}.$$

4.
$$\sum_{n=1}^{\infty} \frac{2n+3}{(n+1)^5 x^{2n}}.$$

5.
$$\sum_{n=1}^{\infty} (-1)^n \frac{(x-2)^{2n}}{2n}.$$

6.
$$\sum_{n=1}^{\infty} \frac{(x-5)^{2n+1}}{3n+8}.$$

$$\begin{array}{lll}
7. \sum_{n=1}^{\infty} \frac{n^3+1}{3^n(x-2)^n} & 8. \sum_{n=1}^{\infty} \frac{n!}{x^n} & 9. \sum_{n=1}^{\infty} \frac{(x+5)^{2n-1}}{4^n(2n-1)} \\
10. \sum_{n=1}^{\infty} \frac{(x-7)^{2n-1}}{(2n^2-5n)4^n} & 11. \sum_{n=1}^{\infty} \frac{(x-2)^n}{(3n+1)2^n} & 12. \sum_{n=2}^{\infty} \frac{3n(x-2)^{3n}}{(5n-8)^3} \\
13. \sum_{n=1}^{\infty} (x+5)^n \operatorname{tg} \frac{1}{3^n} & 14. \sum_{n=1}^{\infty} \sin \frac{\sqrt{n}}{n^2+1} (x-2)^n & 15. \sum_{n=1}^{\infty} \frac{1}{n \cdot 9^n (x-1)^{2n}} \\
16. \sum_{n=1}^{\infty} 3^{n^2} x^{n^2} & 17. \sum_{n=1}^{\infty} \frac{(x+2)^{n^2}}{n^n} & 18. \sum_{n=1}^{\infty} \frac{n^5}{(n+1)!} (x+5)^{2n+1} \\
19. \sum_{n=1}^{\infty} \frac{(3n-2)(x-3)^n}{(n+1)^2 2^{n+1}} & 20. \sum_{n=1}^{\infty} \frac{(x-5)^n}{(n+4) \ln(n+4)} & 21. \sum_{n=2}^{\infty} \frac{1}{(n+2) \ln(n+2) (x-3)^{2n}} \\
22. \sum_{n=5}^{\infty} \frac{1}{2^n n^2 (x+2)^n} & 23. \sum_{n=1}^{\infty} \frac{(x-4)^{n^2}}{n^{n+1}} & 24. \sum_{n=1}^{\infty} \frac{n^5}{x^n} \\
25. \sum_{n=5}^{\infty} \frac{\sqrt{n+1}}{3^n (x+3)^n} & 26. \sum_{n=1}^{\infty} \frac{4^n (x+1)^{2n}}{n} & 27. \sum_{n=1}^{\infty} \frac{3n+5}{(2n+9)^5 (x+2)^{2n}} \\
28. \sum_{n=5}^{\infty} \frac{n^2+1}{5^n (x+4)^n} & 29. \sum_{n=1}^{\infty} \frac{(x+2)^n}{(2n+1)3^n} & 30. \sum_{n=1}^{\infty} \frac{n^2(x-3)^n}{(n^4+1)^2}
\end{array}$$

Задача 19. Разложить функцию в ряд Тейлора по степеням x .

$$\begin{array}{lll}
1. \frac{9}{20-x-x^2} & 2. \frac{x^2}{\sqrt{4-5x}} & 3. \ln(1-x-6x^2) \\
4. 2x \cos^2(x/2) - x & 5. \frac{\operatorname{sh} 2x}{x} - 2 & 6. \frac{7}{12+x-x^2} \\
7. \frac{x}{\sqrt[3]{27-2x}} & 8. \ln(1+x-6x^2) & 9. (x-1) \sin 5x \\
10. \frac{\operatorname{ch} 3x - 1}{x^2} & 11. \frac{6}{8+2x-x^2} & 12. \frac{1}{\sqrt[4]{16-3x}} \\
13. \ln(1-x-12x^2) & 14. (3+e^{-x})^2 & 15. \frac{\operatorname{arcsin} x}{x} - 1 \\
16. \frac{7}{12-x-x^2} & 17. x^2 \sqrt{4-3x} & 18. \ln(1+2x-8x^2) \\
19. 2x \sin^2(x/2) - x & 20. (x-1) \operatorname{sh} x & 21. \frac{5}{6+x-x^2}
\end{array}$$

22. $x\sqrt[3]{27-2x}$.

23. $\ln(1+x-12x^2)$.

24. $\frac{\sin 3x}{x} - \cos 3x$.

25. $\frac{\arctg x}{x}$.

26. $\frac{5}{6-x-x^2}$.

27. $\sqrt[4]{16-5x}$.

28. $\ln(1-x-20x^2)$.

29. $(2-e^x)^2$.

30. $(x-1)\operatorname{ch} x$.

Задача 20. Вычислить интеграл с точностью до 0,001.

1. $\int_0^{0.1} e^{-6x^2} dx$.

2. $\int_0^{0.1} \sin(100x^2) dx$.

3. $\int_0^1 \cos x^2 dx$.

4. $\int_0^{0.5} \frac{dx}{\sqrt[4]{1+x^4}}$.

5. $\int_0^{0.1} \frac{1-e^{-2x}}{x} dx$.

6. $\int_0^1 \frac{\ln(1+x/5)}{x} dx$.

7. $\int_0^{1.5} \frac{dx}{\sqrt[3]{27+x^3}}$.

8. $\int_0^{0.2} e^{-3x^2} dx$.

9. $\int_0^{0.2} \sin(25x^2) dx$.

10. $\int_0^{0.5} \cos(4x^2) dx$.

11. $\int_0^1 \frac{dx}{\sqrt[4]{16+x^4}}$.

12. $\int_0^{0.2} \frac{1-e^{-x}}{x} dx$.

13. $\int_0^{0.4} \frac{\ln(1+x/2)}{x} dx$.

14. $\int_0^2 \frac{dx}{\sqrt[3]{64+x^3}}$.

15. $\int_0^{0.3} e^{-2x^2} dx$.

16. $\int_0^{0.4} \sin(5x/2)^2 dx$.

17. $\int_0^{0.2} \cos(25x^2) dx$.

18. $\int_0^{1.5} \frac{dx}{\sqrt[4]{81+x^4}}$.

19. $\int_0^{0.4} \frac{1-e^{-x/2}}{x} dx$.

20. $\int_0^{0.1} \frac{\ln(1+2x)}{x} dx$.

21. $\int_0^{2.5} \frac{dx}{\sqrt[3]{125+x^3}}$.

22. $\int_0^{0.4} e^{-3x^2/4} dx$.

23. $\int_0^{0.5} \sin(4x^2) dx$.

24. $\int_0^{0.4} \cos(5x/2)^2 dx$.

25. $\int_0^2 \frac{dx}{\sqrt[4]{256+x^4}}$.

26. $\int_0^{0.5} \frac{dx}{\sqrt[3]{1+x^3}}$.

27. $\int_0^{2.5} \frac{dx}{\sqrt[4]{625+x^4}}$.

28. $\int_0^1 \frac{dx}{\sqrt[3]{8+x^3}}$.

29. $\int_0^{0.5} e^{-3x^2/25} dx$.

30. $\int_0^1 \sin x^2 dx$.